Investment Strategies with VIX and VSTOXX

Silvia Stanescu

Kent Business School,

University of Kent, Canterbury,

CT2 7PE, s.stanescu@kent.ac.uk

Radu Tunaru

Kent Business School,

University of Kent, Canterbury

CT2 7PE, r.tunaru@kent.ac.uk

Abstract

VIX and VSTOXX derivatives have been the story of success in terms of product innovation over the last five years. In this paper we use historical data on S&P500 and EURO STOXX 50, VIX and VSTOXX, and VIX and VSTOXX Futures to reveal linkages between these important series that can be used by equity investors to generate alpha and protect their investments during turbulent times. We consider for comparative performance purposes investment portfolios in U.S. and EU zone and also a long-short cross border portfolio. The econometric analysis is spanned by a battery of GARCH models from which we have selected the GARCH (1,1), the EGARCH and the GJR model as the best models for our data. Overall, investors with EURO STOXX 50 exposure can improve greatly the performance of their portfolio by adding VSTOXX futures.

Investment Strategies with VIX and VSTOXX

1. Introduction

1.1 *Background*

"The CBOE Volatility Index (VIX) is a key measure of market expectations of near-term volatility conveyed by S&P500 stock index option prices. Since its introduction in 1993, VIX has been considered by many to be the world's premier barometer of investor sentiment and market volatility." – Website of CBOE

Likewise, the VSTOXX index is also a volatility index, based on the expected volatility implied by EURO STOXX 50 options. There are 12 VSTOXX rolling indices with maturities equal to 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 330 and 360 days to expiration. The calculation is done via linear interpolation of the two nearest subindices. Each of the 8 sub-indices per option expiry (1, 2, 3, 6, 9, 12, 18 and 24 months) is determined based on the square-root of the implied variance.

The main attraction of the VIX and VSTOXX products lies in the negative correlation of these volatility indices with the corresponding equity market indices, usually explained by the leverage effect[1]. The evolution of S&P500 and VIX illustrated in Figure 1 and, respectively of EURO STOXX 50 and VSTOXX in Figure 2 seems to support the idea of a negative correlation, implying that adding VIX and VSTOXX positions (via futures contracts) would help in reducing the risk of diversified portfolios. This connection helped the growth of the volatility derivatives market to the extent that many investors perceive VIX and VSTOXX as an asset class of its own.

The two graphs also indicate that there is a shock event in the equity space every time the volatility index crosses the corresponding equity index. One could then differentiate between the usual market jumps in volatility due to changes in policy, board announcements and takeover

[1] Any fall in equity prices leads to an increase in the company's leverage, which in turn increases the risk posed to equity holders and therefore increases equity volatility. On the contrary, a decrease in equity prices leads to reduced leverage and then the risk posed to equity holders is reduced and equity volatility becomes smaller.

attempts, and the jumps directly related to crashes of significant importance such as Lehman Brothers in September 2008.

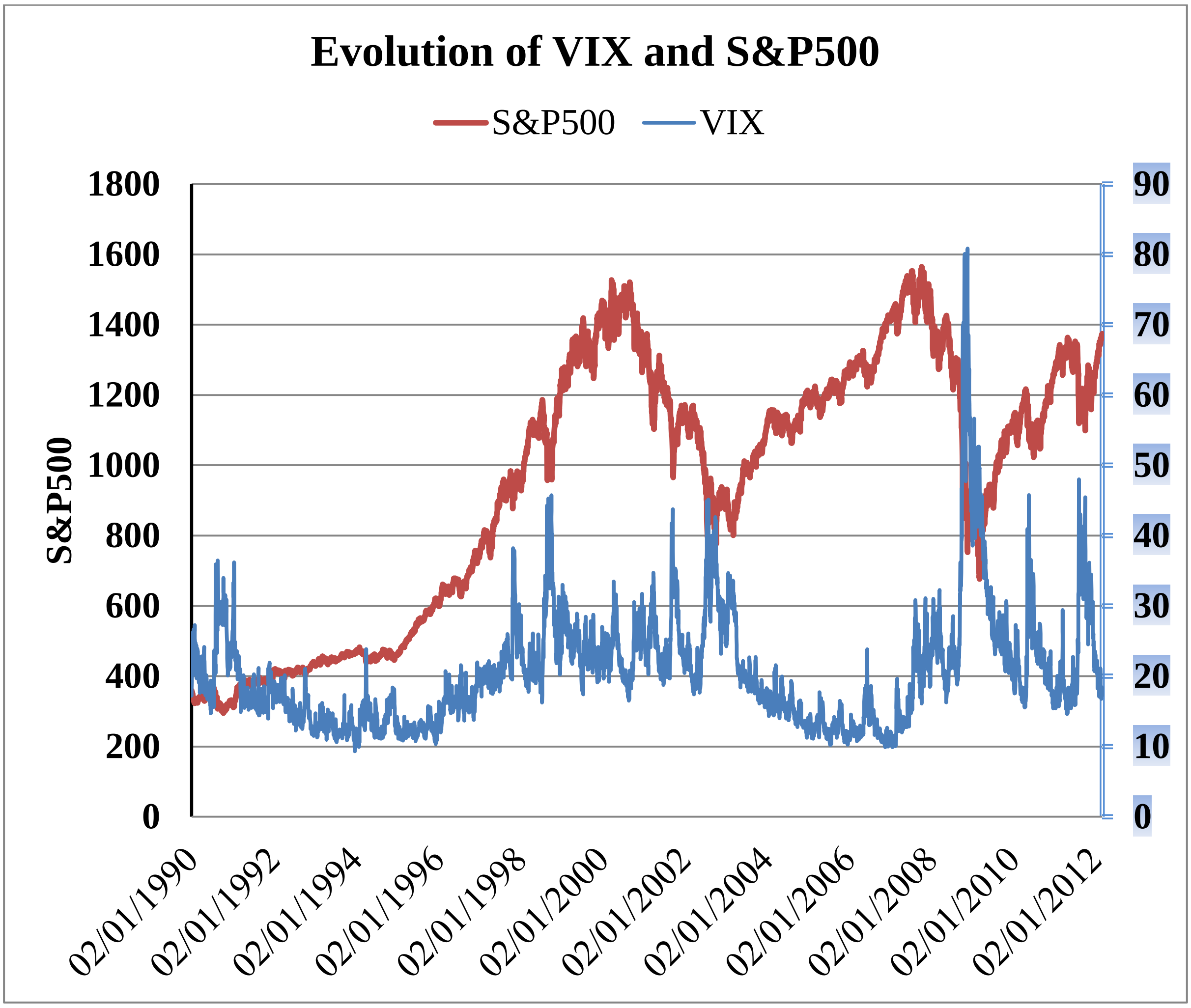

Figure 1. Daily time series of S&P500 and VIX between 02-01-1990 and 01-03-2012.

However, a very important question is the degree of correlation that is revealed by the data. Recall that the correlation concept that is usually invoked in this context is the Pearson linear correlation coefficient, for which we know that a value of 1 or -1 is equivalent with a linear relationship between the two variables. As it can be observed from Figures A1 and A2 from Appendix B there is indeed evidence of a linear decreasing relationship for the series of logarithmic returns of the equity index and the corresponding volatility index but the gradient of the line fitted to the historical data is not -1. It is also clear that the daily returns for both equity indexes are between -10% and 10% whereas the returns for the volatility indexes are roughly speaking between -30% and 40% for VIX, and between -20% and 35% for VSTOXX.

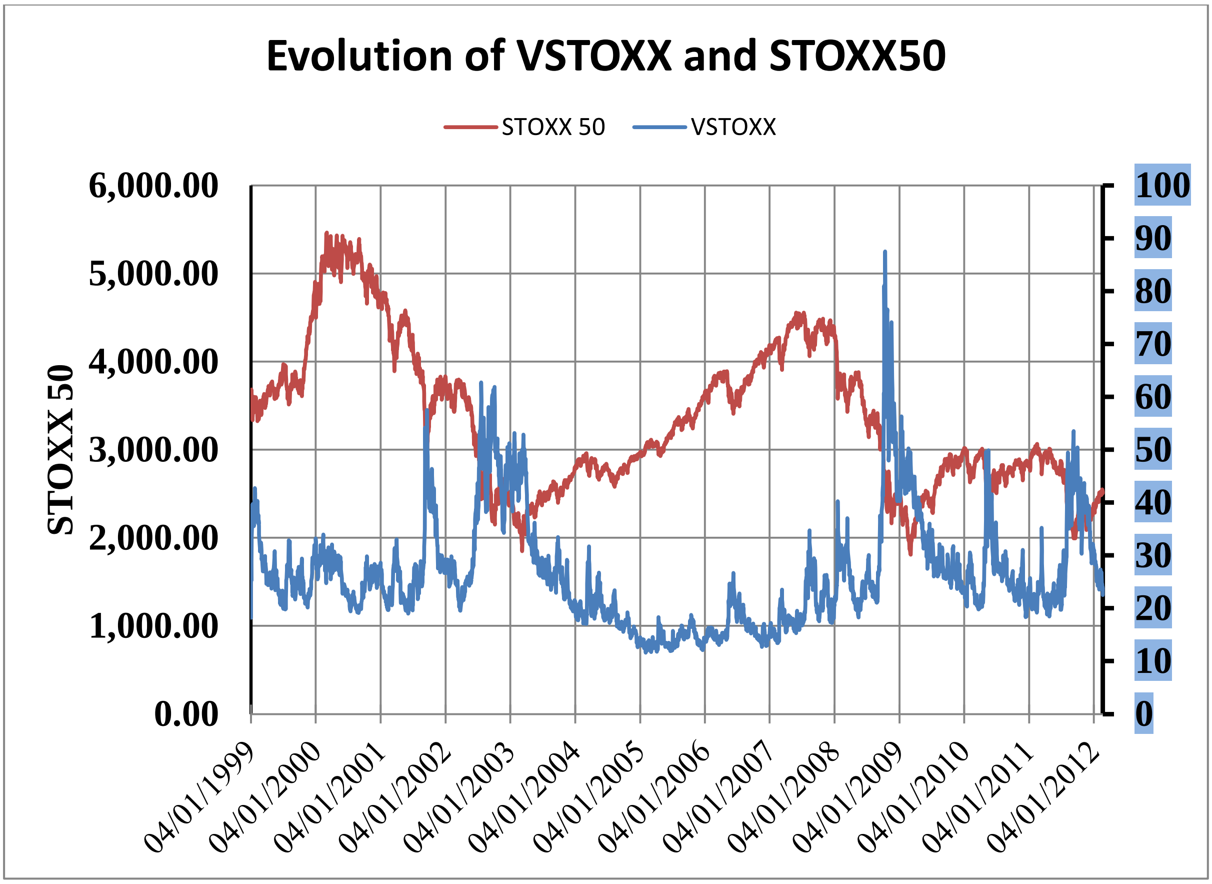

Figure 2. Daily time series of EURO STOXX50 and VTOXX between 04-01-1999 and 24-02-2012.

Including volatility positions in an investment portfolio can be done either for portfolio diversification or for hedging purposes. The latter is true for portfolio managers that are tracking index equity portfolios and who are short volatility. When equity markets become highly volatile then the portfolio tracking error and the rebalancing costs increase but using volatility futures helps to hedge against these frictional costs. At the other extreme, the volatility futures contracts offer a direct play on the vega with no delta involved. Hence, speculative directional positions can be taken via VIX and VSTOXX futures. An interesting trading strategy is based on the correlation between the VSTOXX and VIX. A fund manager may buy be long VSTOXX volatility and short VIX volatility. A similar idea is to trade on the basis between VIX and VSTOXX, given the historical evolution between the two.

The body of this paper is structured as follows: the following two sub-sections describe in some detail the VIX and VIX futures contracts and VSTOXX and VSTOXX futures contracts, respectively. Section 2 reviews the existing literature on volatility indices, while Sections 3 and 4 focus on data, methodology and empirical results. In Section 5 some investment strategies based on the findings in this paper are implemented and discussed. The final section puts forth a number of recommendations and conclusions.

1.2 *VIX and VIX Futures*

The VIX index has been introduced by Whaley (1993) and the methodology was further revised in 2003. This index measures the market's implied view of future volatility of the equity S&P500 index, given by the current S&P 500 stock index option prices[2]. When constructing the VIX, the put and call options are near- and next-term, usually in the first and second S&P500 contract months. "Near-term" options must have at least one week to maturity. This condition is imposed in order to minimize pricing anomalies that might appear close to expiration. When this condition is violated VIX "rolls" to the second and third S&P500 contract months[3].

It is important to realize that the VIX is a measure of expected future volatility but it also incorporates the uncertainty on the market triggered by various bank crashes and crises. In Figure 3 we show the VIX levels versus the contemporaneous realized volatility on the S&P500 index. Simon (2003) argued that market participants tend to consider extreme values of VIX as trading signals. Looking at the peaks of the realized variance, VIX is always under, predicting that the realized levels of volatility during market turbulence were unsustainable. Although the above example suggest that VIX is an accurate predictor of falling volatility, a more thorough analysis is needed in order to draw such an important conclusion.

Considering the evolution of the VIX index depicted in Figure 3 it can be seen that it was relatively stable in the early 1990s, but started to be "volatile" from the last quarter of 1997 to the first quarter of 2003. Another clear milestone was the end of the year 2007 associated with the burst of the subprime crisis leading to spikes in the values of the VIX. The spikes in the time series of the VIX can be pinpointed to the Iraq war in early 1991, the Asian financial crisis of late

[2] The CBOE changed the composition of the VIX on September 22, 2003. For the period January 2, 1986, to September 19, 2003, the VIX was calculated from S&P 100 index option prices. From September 22, 2003, the calculation of VIX has been changed to S&P 500 index option prices. It can be argued that, since the S&P 100 and S&P 500 index portfolios are very similar, using the VIX history based on S&P 100 prices until September 22, 2003 (i.e. the cleaner, more accurate historical series), and then the VIX history based on S&P500 option prices, is an acceptable way to put together a historical VIX time series. The current methodology is independent of a pricing model, VIX being calculated in practice from market option prices. CBOE recalculated the VIX values under the current methodology from January 1, 1990.

[3] For example, on the second Friday in June, VIX should be determined from S&P500 options expiring in June and July. On the following Monday, July maturity will replace June as the "near-term" and August maturity will replace July as the "next-term."

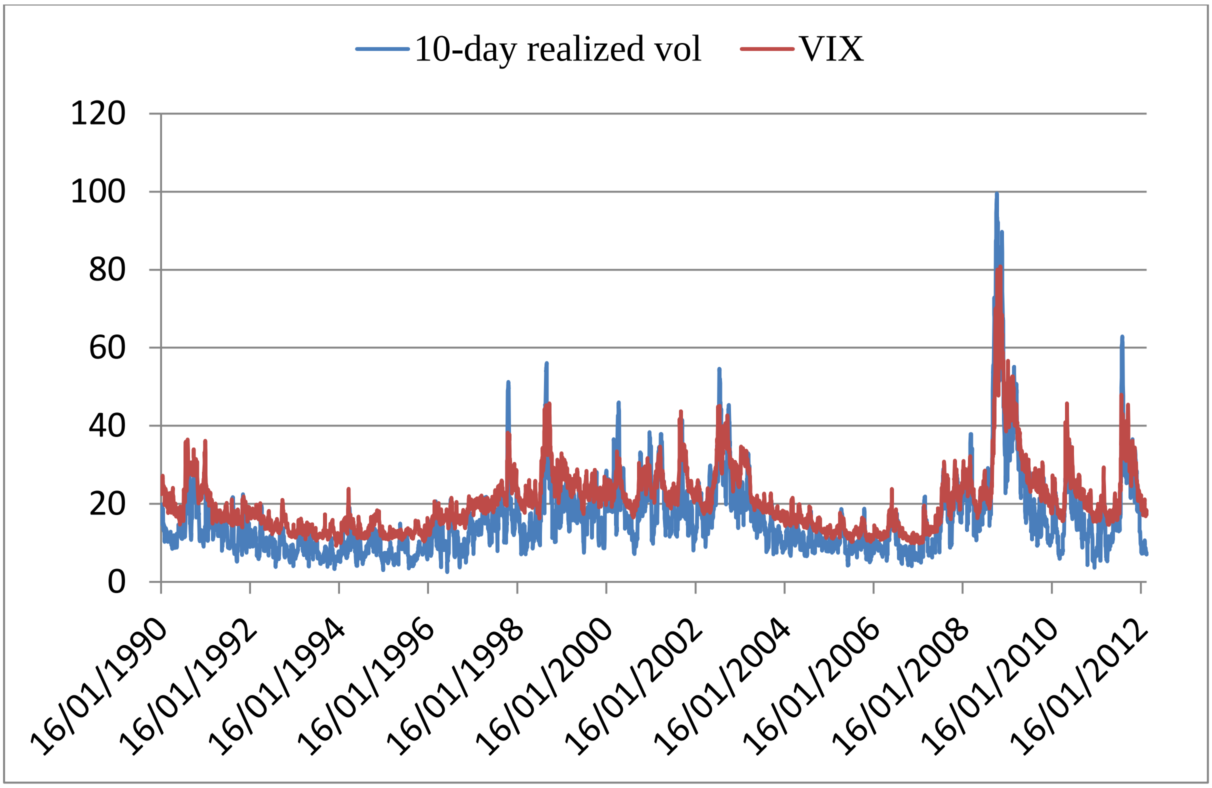

Figure 3. Comparison of time series of VIX, calculated under the post 2003 methodology, with the historical 10-day realized volatility, on the same day. The period covered is 16-01-1990 and 01-03-2012.

1997, the Russian and LTCM crisis of late summer 1998, and the 9/11 terrorist attacks. The post 2007 spikes are associated with the Lehman Brothers collapse of 2008 and the emergence of the sovereign debt problems in Euro zone in 2010.

Futures contracts on VIX have started trading on 26 March 2004 and options in February 2006. A Mini-VIX futures contract has been launched in 2009.

1.2 *VSTOXX and VSTOXX Futures*

The EURO STOXX 50 Index is constructed from Blue-chip companies of sector leaders in the Eurozone: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. The EURO STOXX 50 Volatility Index (VSTOXX) index provides the implied volatility given by the prices of the options with corresponding maturity, on EURO STOXX 50 Index. By design the VSTOXX index is based on the square root of implied variance and it calibrates the volatility skew from OTM puts and calls. The VSTOXX does not measure implied volatilities of at-the-money EURO STOXX 50 options, but the implied variance across all options of a given time to expiry. This model has been jointly developed by

Goldman Sachs and Deutsche Börse such that using linear interpolation of the two nearest sub-indices, a rolling index of 30 days to expiration is calculated every 5 seconds using real-time EURO STOXX50 option bid/ask quotes. The VSTOXX is calculated on the basis of eight expiry months with a maximum time to expiry of two years[4]. If there are no such surrounding sub-indices, nearest to the time to expiry of 30 days, the VSTOXX is calculated using extrapolation, using the two nearest available indices which are as close to the time to expiry of 30 calendar days as possible. In the situation that there are no two such indices VSTOXX is calculated by extrapolation based on the nearest available indices, which are as close to 30 calendar days as possible.

The payoff of VSTOXX futures resembles more the payoff of a volatility swap, being determined by the difference between the realized 30 day implied volatility and the expected 30 day implied volatility at trade initiation, times the number of contracts and the monetary size of the index multiplied (€100).

The VSTOXX Short-Term Futures Index is designed to replicate the performance of a long position in constant-maturity one-month forward, one-month implied volatilities on the EURO STOXX 50. Similarly, the VSTOXX Mid-Term Futures Index replicates a constant 5-month forward, one-month implied volatility. The VSTOXX Short-Term Futures index aims to provide a return of a long position in constant maturity one-month forward one-month implied volatilities on the underlying EURO STOXX 50 Index. In addition, another EURO STOXX 50 Index future contract has been launched in December 2010 on the Singapore Exchange, enabling investors to react to Asian market developments and trade the EURO STOXX 50 before the opening of the European markets. This is a quanto type contract with a value of USD 10 per index point.

The graph in Figure 4 reveals the same type of conclusion as in the VIX case, that is the levels exhibit by the realized volatility during market turbulence are not sustainable and in the short term volatility will decrease.

[4] Apart from the VSTOXX main index (which is irrespective of a specific time to expiry), sub-indices for each time to expiry of the EURO STOXX 50 options, ranging from one month to two years, are calculated and distributed. For options with longer time to expire, no such sub-indices are currently available.

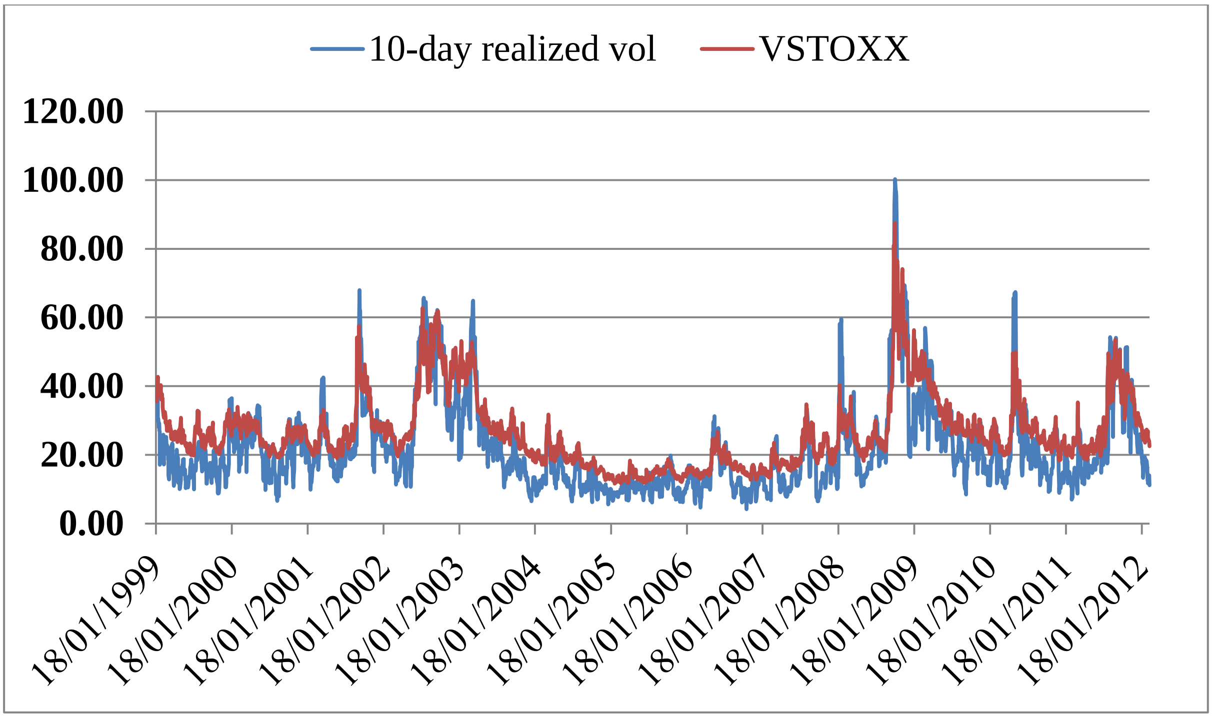

Figure 4. Comparison of time series of VSTOXX with the historical 10-day realized volatility, on the same day. The period covered is 18-01-1999 and 24-02-2012.

2. Literature Review

2.1 *The Relationship Between Implied Volatility and the Future Realized Volatility*

The question how well the implied volatility forecasts future realized volatility has been received a great deal of attention in the financial literature, the general conclusion being that implied volatility outperforms the known historical volatility measures, see Fleming (1995), Blair et.al. (2001), Corrado & Miller (2005). Becker et.al. (2006), however, found that VIX is not an efficient forecaster of future realized volatility and other historical volatility estimates can be superior to VIX alone.

2.2 *The Relationship Between Implied Volatility and Stock Returns*

Whaley (2000) was among the first to point out that there is a negative statistically significant relationship between the returns of stock and associated implied volatility indexes and moreover, positive stock index returns correspond to declining implied volatility levels, while negative returns correspond to increasing implied volatility levels. For the S&P 100 index, the relationship is asymmetric, negative stock index returns are triggered by greater proportional changes in implied volatility measures than are positive returns.

Carr and Wu (2006) argued that it is the S&P 500 index returns that predict future movements in the volatility index VIX and that volatility index movements do not have predictive power on the equity index returns. On the other hand, Cipollini and Manzini (2007), using the same methodology as in Giot (2005) and Campbell and Shiller (1998), identified a significant relationship between the VIX levels and the 3-months S&P 500 Index returns. This linkage is very strong following spikes in VIX while it is weaker at lower levels of VIX. Their trading strategy to invest in the S&P5 00 index based on the VIX signal outperforms the simple strategy of holding long the S&P 500 index, confirming wide spread belief in investment banking.

Konstantinidi et al. (2008) discussed several models for implied volatility indexes including the VIX showing that the directional change can be forecasted using point and interval forecasts. The directional forecast accuracy can be improved by using GARCH models as demonstrated in Ahoniemi (2008). Compared with various standard time series models, an ARIMA(1,1,1) model with GARCH errors fits the historical VIX data well in this study, the directional accuracy of forecasts being close to 60% over a five year out-of-sample period. One major point made by Ahoniemi (2008) is that the addition of GARCH errors contributes significantly to forecast performance while the inclusion of S&P 500 returns in the model does not improve the directional forecasts. This is in line with Christoffersen and Diebold (2006), who demonstrate that it is possible to predict the direction of change of returns in the presence of conditional heteroskedasticity, even if it is not possible to predict the returns themselves.

Banerjee et al.(2007) and Giot (2005) develop models that use the VIX to predict stock market returns. The latter investigates the link between contemporaneous relative changes in VIX and contemporaneous S&P500 returns, but also the relationship between the current VIX levels and the future stock index returns. Denoting VIX_t the value of VIX index and by OEX_t the value of S&P100 index at time *t*, then $r_{VIX,t} = \ln(VIX_t / VIX_{t-1})$ and $r_{OEX,t} = \ln(OEX_t / OEX_{t-1})$ are the logarithmic returns of the two indexes, then Giot (2005) fitted the regression

$$r_{VIX,t} = \beta_0^+ D_t^+ + \beta_0^- D_t^- + \beta_1^+ r_{OEX,t} D_t^+ + \beta_1^- r_{OEX,t} D_t^- + \varepsilon_t \tag{1}$$

where D_t^- is a dummy variable that is equal to 1 (0) when $r_{OEX,t}$ is negative (positive) and $D_t^+ = 1 - D_t^-$. Based on this regression Giot concluded that negative returns for the stock index are associated with much greater relative changes in the implied volatility index than are positive returns.

Whaley (2009) discussed the observed VIX spikes during market unrest. He noted that when market volatility increases or decreases, respectively, the stock prices will fall, or rise respectively. The relationship between the rate of change on VIX and the rate of return on the corresponding S&P500 index (SPX) is more than one of proportionality and he argues that the change in VIX should rise quicker when the market falls than when the market rises, in line with the leverage argument proposed by Black. This hypothesis is tested using the following regression model

$$r_{VIX,t} = \beta_0 + \beta_1 r_{SPX,t} + \beta_2 r_{SPX,t} D_t^- + \varepsilon_t \quad (2)$$

Szado (2009) showed that adding VIX futures during the 2008 financial crisis to three base portfolios resulted in increased returns and reduced standard deviations. It was shown in the paper that when adding ATM VIX calls to the three base portfolios will increase portfolio returns but the effect on standard deviation was mixed, with more extreme results, not surprisingly given the extra leverage. Using VIX call options increased the profits during market drops but correspondingly also increased the standard deviation. The comparative analysis of buying S&P500 puts with the three base portfolios did not produced better results than when adding VIX Call options. Similarly, Chen et.al. 2011 demonstrated that adding VIX futures contracts can improve the mean-variance investment frontier so hedge fund managers for example may be able to enhance their equity portfolio performance, as measured by the Sharpe ratio.

2.3 *The Relationship between Implied Volatility Index and Its Futures Contract*

Brenner et.al (2007) showed that the term structure of VIX futures price is upward sloping while the term structure of VIX futures volatility is downward sloping. Dash and Moran (2005) discussed the advantages of using VIX as a companion for hedge fund portfolios.

3. Portfolio Diversification with VIX and VSTOXX

3.1 *Portfolio diversification with futures*

The theoretical argument tells us that, absent any market frictions, whenever we have a hedge instrument written on the same underlying as our original exposure and with maturity matching our hedge horizon a perfect hedge is possible. However, we are often in a situation where proxy hedges (i.e. hedges on a different, but related underlying to the original exposure) are used. This could be for liquidity, cost or other reasons.

Preliminary Analysis - Correlations

In this subsection we compare the diversification effectiveness with VSTOXX vs. VIX-related instruments. As the effectiveness of the hedge will depend on the correlation between the original exposure and the hedge, we first consider the correlations between the EURO STOXX50 and VSTOXX daily log returns and between S&P 500 and VIX returns. We expect to find negative correlations between the returns on the two equity indices and those on their respective volatility indices. Figure 3.1 plots the 30-day historical correlations for these two pairs of variables, while Figure 3.2 compares the same 30-day historical correlations between EURO STOXX 50 returns and VIX and VSTOXX returns, respectively.

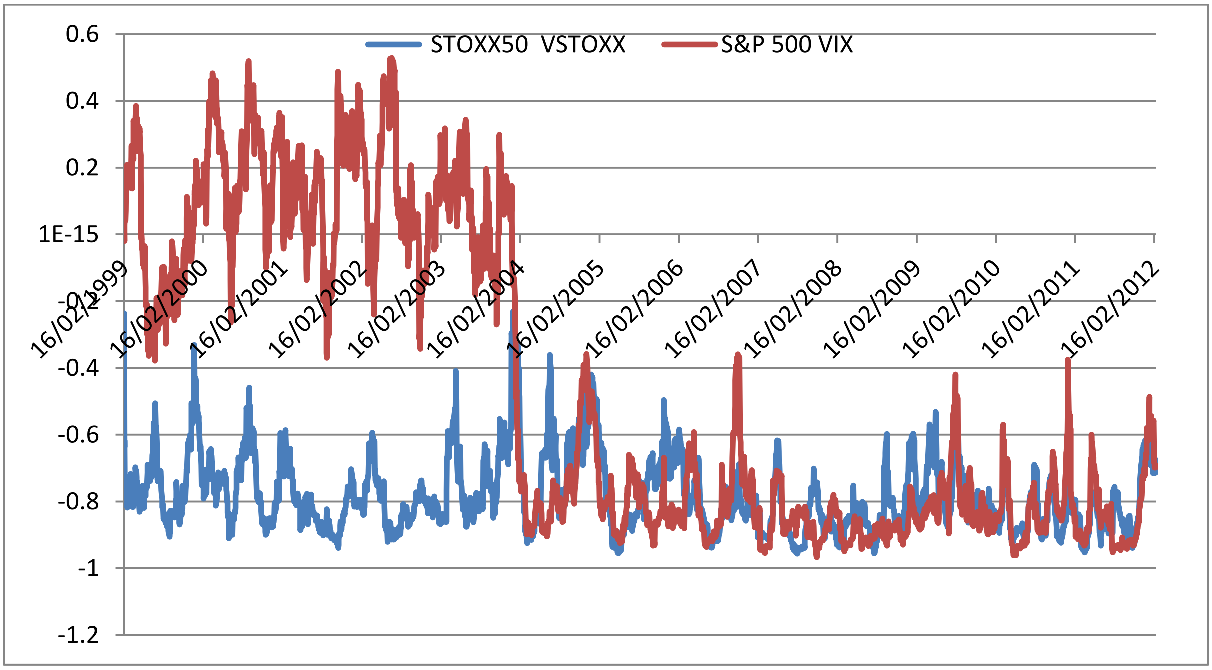

Figure 3.1 30-day Historical Correlations: S&P 500 vs. VIX and EURO STOXX vs. VSTOXX

Note: The correlations are computed for the daily log returns; each correlation estimate is based on the 30 working day sample pre-dating it.

It is easily noticeable from these two figures that while the correlations between the EURO STOXX50 and VSTOXX are always negative, the correlations between S&P 500 and VIX are positive for part of the sample. We note that the period under consideration is January 1999 to

January 2012 and we recall that for the first part of the sample (i.e. January 1999 to 19th September 2003) the VIX was calculated based on the implied volatility of S&P 100 options. Therefore it is not surprising that for the period predating September 2003 the correlation between the S&P 500 returns and VIX is not so strongly negative, since for this period the VIX was actually based on a different index. It is worthwhile noting that for this period the VIX would be expected to prove a less efficient diversifier for a portfolio that tracks the S&P 500 since, for the period to 22nd September 2003, the VIX calculation was based on the implied volatility of different index. The same argument applies to the use of the VIX as diversifier for portfolios resembling the EURO STOXX50. As it can be noticed from Figure 3.2, the 30-day historical correlation between the EURO STOXX 50 and VIX takes positive values for some of the sample days prior to 2006; also, while the correlation between EURO STOXX 50 and VIX is always negative post 2006, it is less so than the correlation between EURO STOXX 50 and VSTOXX. Moreover, the correlation between the EURO STOXX 50 and VSTOXX remains negative throughout the entire sample. Thus, the VSTOXX volatility index appears to be a more efficient diversifier for EURO STOXX investors that the VIX.

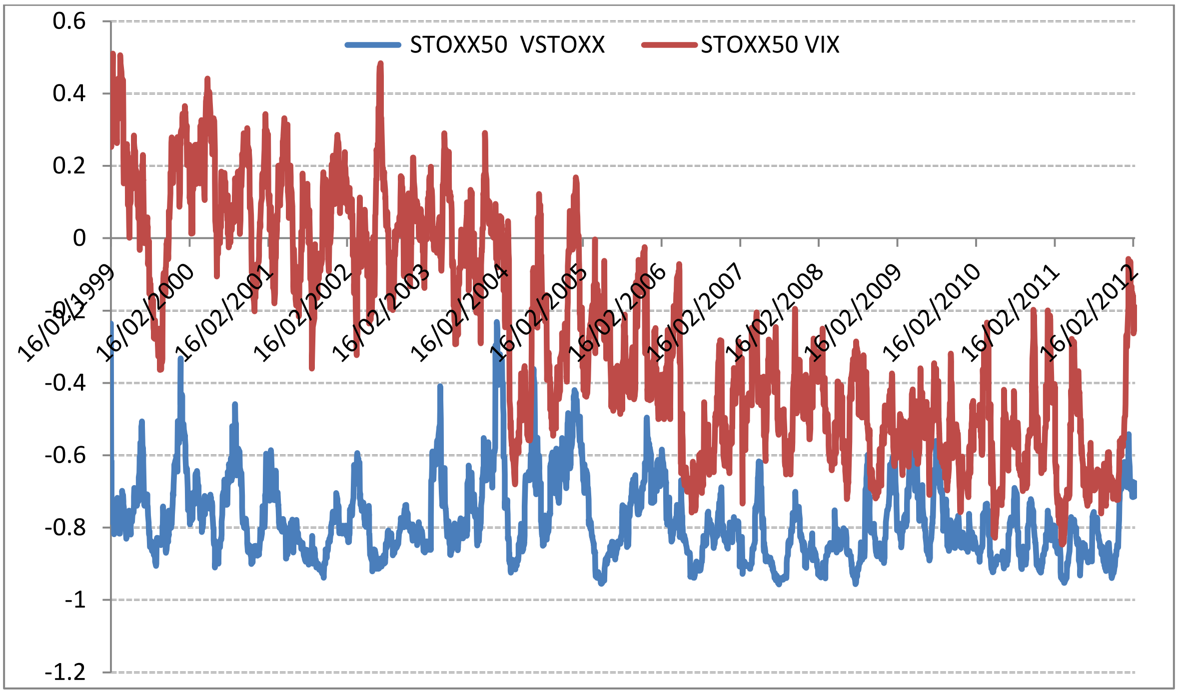

Figure 3.2 30-day Historical Correlations: EURO STOXX50 vs. VSTOXX and VIX

Note: The correlations are computed for the daily log returns; each correlation estimate is based on the 30 working day sample pre-dating it.

However, since the VIX and VSTOXX volatility indices are not investable instruments, in Figures 3.3. and 3.4 we consider the correlation between the daily log returns on the equity indices (S&P 500 and EURO STOXX 50) and the VIX and VSTOXX daily log returns.[5] The nearest maturity futures contract is considered in both of these graphs. We note that correlations between the two equity indices and their respective volatility index futures returns remain negative throughout the sample periods considered; however, returns on the indices appear to be less correlated (i.e. the absolute value of correlations is lower) with the returns on the nearest maturity volatility index futures than with the returns on the respective volatility index.

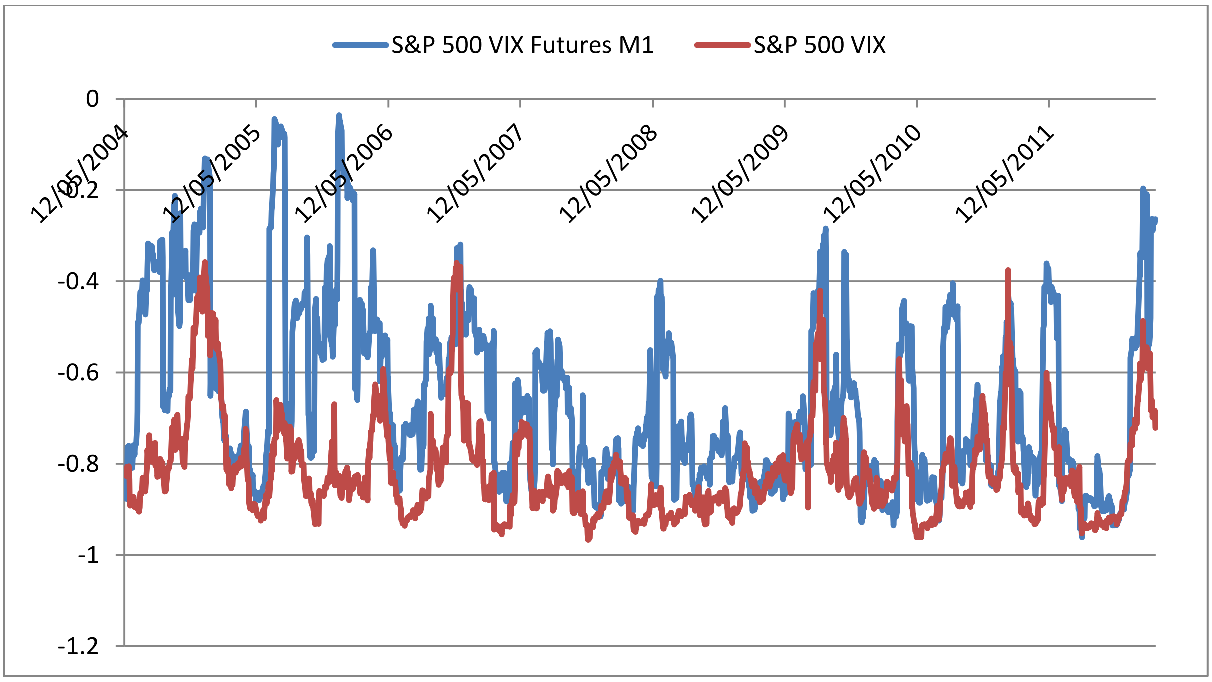

Figure 3.3 30-day Historical Correlations: S&P 500 vs. VIX and VIX Futures

Note: The correlations are computed for the daily log returns; each correlation estimate is based on the 30 working day sample pre-dating it.

[5] VIX futures were introduced in 2004 and VSTOXX futures in 2009, hence Figures 3.3 and 3.4 only plot correlations for samples starting in 2004 and 2009, respectively.

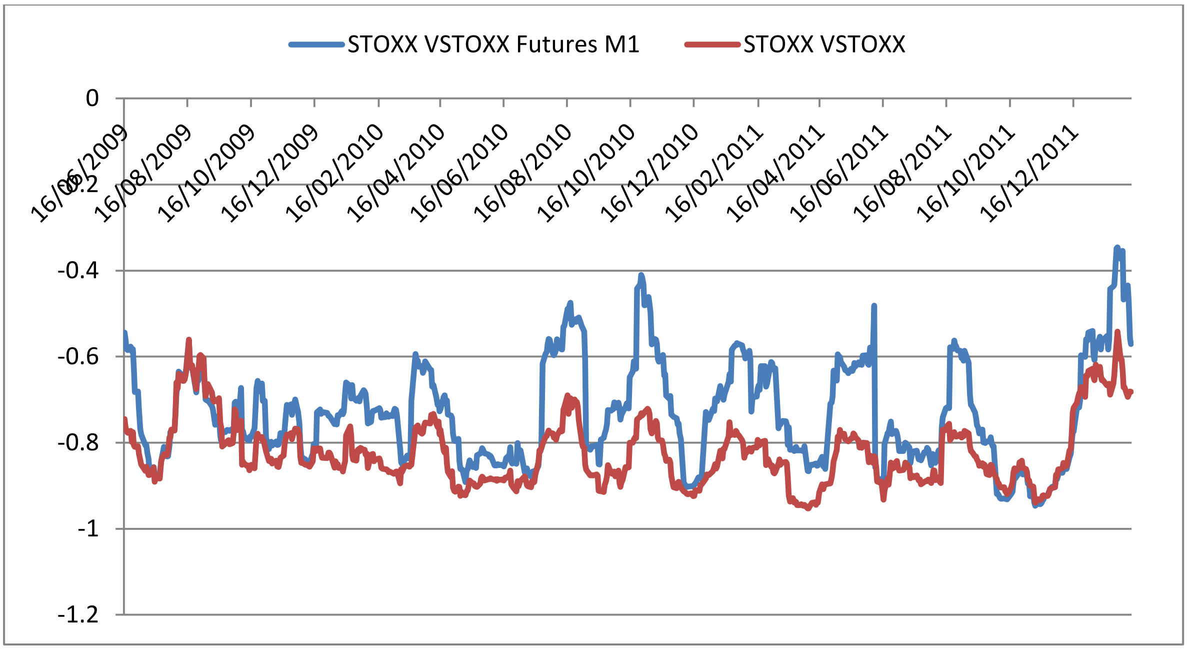

Figure 3.4 30-day Historical Correlations: EURO STOXX 50 vs. VSTOXX and VSTOXX Futures

Note: The correlations are computed for the daily log returns; each correlation estimate is based on the 30 working day sample pre-dating it.

Rhoads (2011) notes that using only the front month VIX futures contract in a diversified portfolio can be sub-optimal) in the long term (high costs, underperformance in bullish markets and overall underperformance in the long term and suggests using the front two months contracts. We therefore also plot in Figures 3.5 and 3.6 the correlation between the two equity indices under consideration and the second nearest maturity contract. We note that while the correlation between S&P 500 and the nearest maturity VIX futures was always negative, the correlation between the equity index returns and the second maturity VIX futures takes a few positive, albeit very small values in the first part of the sample. However, as Rhoads (2011) also notes, this could be due to the lighter trading of the contract in its early days – post 2007 the correlations with the second maturity futures returns are always negative. The correlations between the daily returns on the EURO STOXX 50 index and the daily returns on the VSTOXX (spot) and VSTOXX futures, both nearest and second nearest maturities (Figure 3.6) remain negative throughout the entire sample.

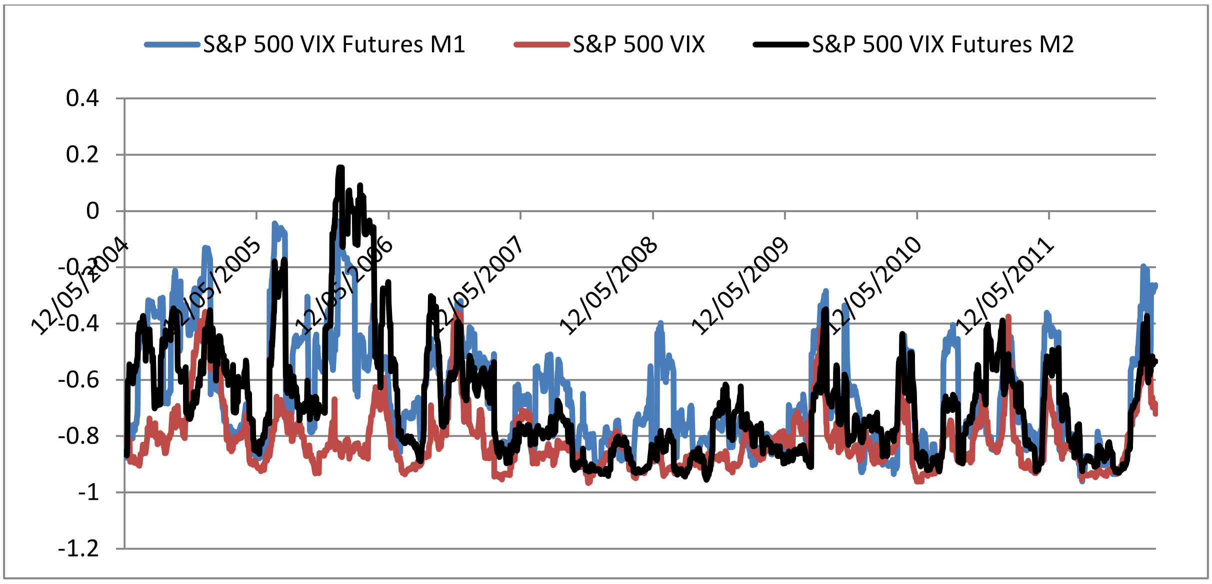

Figure 3.5 30-day Historical correlations: S&P 500 vs. VIX and VIX futures

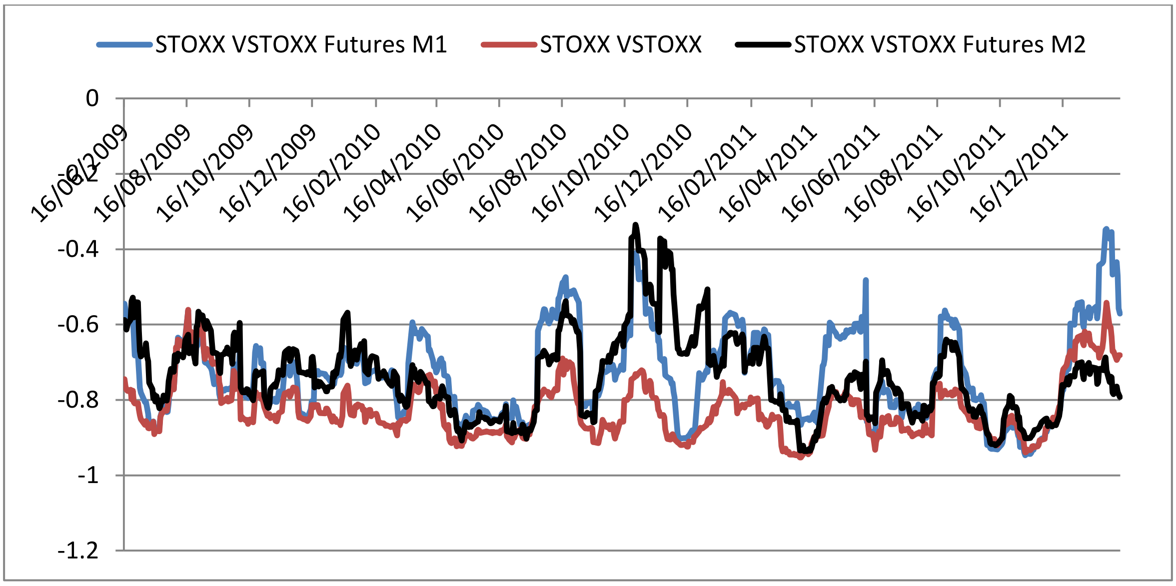

Figure 3.6 30-day Historical Correlations: EURO STOXX 50 vs. VSTOXX and VSTOXX futures

To eliminate any influences coming from approaching time to maturity, we could construct a portfolio consisting of the two nearest maturities futures contracts, which each have dynamic weights linked to their remaining time to maturity: the closer the maturity, the lower the weight the respective future contract has. The fact that the dynamics of VIX and VSTOXX is not replicated closely by their futures contracts is in line with the conclusions in Moran and Dash (2007).

3.2 *Portfolio performance with volatility diversification*

Following Szado (2009), for each of the volatility indices (i.e. VIX and VSTOXX) we consider the following portfolios which will be compared relative to the shocks in volatility:

1. 100% equity – we will assume that the investor holds a portfolio that tracks the S&P 500 or the EURO STOXX 50 indices, respectively.
2. 60% equity + 40% bonds, where the bond exposure will be represented by a portfolio that resembles the Barclays US or Barclays EURO Total Return Indices, respectively

A set of summary statistics for all the components of these portfolios as well as for the hedge instruments proposed below (i.e. VIX and VSTOXX futures) is given in Tables 3.1 and 3.2 below. For the US Sample, the data ranges from March 2004 (when the VIX futures were introduced) to February 2012. By contrast, the European sample is shorter, since VSTOXX futures were only introduced at the end of April 2009. The US sample is split into two sub-periods: a pre-crisis period (2004-2007) and a post-crisis period (2008-2012). We also analyze the returns of 2008 separately, as this is the period in which markets saw the most dramatic movements. As expected the volatility of the volatility-related assets, namely VIX and VSTOXX futures, is highest and the volatility of the bond indices is lowest; this is true for both samples (US and Europe) and for all sub-periods considered (in the US case). The range of returns is also widest for the volatility related assets, which exhibit both the highest maximums and the lowest minimums, again across both samples and all sub-periods. By contrast, bonds have the narrowest ranges of returns.

	S&P500	**Bond Index**	**VIX first maturity**	**VIX second maturity**
Annualized mean return	2.63%	5.16%	1.36%	2.65%
Volatility	22.27%	3.99%	79.61%	53.79%
Min	-9.47%	-1.26%	-29.48%	-18.57%
Max	2.13%	0.91%	36.02%	13.04%
Skewness	-0.2859	-0.0516	0.9363	0.6234
Excess Kurtosis	9.7162	1.7630	5.6505	3.3574
subperiod 1: 2004 - 2007				
Annualized mean return	7.57%	4.05%	3.47%	4.90%
Volatility	12.10%	3.27%	70.54%	45.31%
Min	-3.53%	-0.98%	-29.48%	-15.38%
Max	2.88%	0.91%	36.02%	14.45%
Skewness	-0.3205	-0.0393	1.4064	0.8376
Excess Kurtosis	1.9553	1.5829	11.9821	5.0127
subperiod 2: 2008-2012				
Annualized mean return	-1.85%	6.17%	-0.56%	0.61%
Volatility	28.53%	4.55%	87.08%	60.51%
Min	-9.47%	-1.26%	-23.13%	-18.57%

Max	10.96%	1.33%	23.57%	17.00%
Skewness	-0.2133	-0.0736	0.6899	0.5207
Excess Kurtosis	5.8829	1.2839	2.7090	2.3143
crisis subperiod: 2008				
Annualized mean return	-50.80%	5.47%	65.80%	63.25%
Volatility	41.41%	5.93%	94.17%	60.86%
Min	-9.47%	-1.26%	-23.13%	-18.57%
Max	10.96%	1.24%	23.57%	12.82%
Skewness	-0.021	-0.1278	0.0069	0.0323
Excess Kurtosis	3.6484	0.4911	2.8567	2.3229

Table 3.1 Summary Statistics of log returns series for the portfolio components of U.S. Market

Notes: *The summary statistics are of the daily returns on the S&P 500 equity index, Barclays US Aggregated total return bond index from 26h March 2009 to 17th February 2012. The standard errors are approximately* $(6/T)^{1/2}$ *and* $(24/T)^{1/2}$ *for the sample skewness and excess kurtosis, respectively, where T is the sample size. The values of the t statistic for both the sample skewness and excess kurtosis indicate that returns for most of the assets considered follow non-normal distributions, generally leptokurtic.*

	Euro STOXX 50	**Bond Index (EUR)**	**VSTOXX Futures M1**	**VSTOXX Futures M2**
Annualized mean return	2.17%	4.65%	-12.85%	-9.41%
Volatility (annualized st dev)	25.49%	3.22%	77.76%	51.73%
Min	-6.54%	-0.78%	-17.38%	-12.57%
Max	9.85%	1.08%	21.22%	12.35%
Skewness	0.0375	0.4037	0.7393	0.3868
t-statistic Skewness	0.4036	4.3480	7.9620	4.1663
Excess Kurtosis	3.0642	3.1098	2.5495	1.4323
t-statistic Kurtosis	16.5014	16.7469	13.7297	7.7130

Table 3.2 Summary statistics of log returns series for the portfolio components: European Market

Notes: *The summary statistics are of the daily returns on the EURO STOXX 50 equity index, Barclays EURO Aggregated total return bond index from 30th April 2009 to 9th February 2012. The standard errors are approximately* $(6/T)^{1/2}$ *and* $(24/T)^{1/2}$ *for the sample skewness and excess kurtosis, respectively, where T is the sample size. The values of the t statistic for both the sample skewness and excess kurtosis indicate that returns for all the assets considered follow non-normal distributions, all of them leptokurtic.*

The returns distributions are generally non-normal: with the exception of US bonds in the 2008 sub-period, all the other returns distributions exhibit positive and highly significant (t-statistics higher than 7) values of the excess kurtosis. As expected, equity index returns are generally negatively skewed, while volatility futures returns exhibit positive skewness.

We now turn to the construction and analysis of the volatility-diversified portfolios. Following Szado (2009), we pre-set the portfolio weights for the volatility futures to 2.5% and then 10%.

We shall relax this assumption in following sections where we shall consider alternative methods of (optimally) determining the level of these portfolio weights.

Tables 3.3 and 3.4 summarize the performance of the volatility-diversified portfolios. We assume that the portfolios are rebalanced weekly. In order to be able to compute the Sharpe ratios reported in these tables, we use the 3-months Treasury Bill rates (secondary markets) in place of the risk free rate for the US portfolio. We employ the 3-month EURO LIBOR rate as the EURO risk free rate.[6] The results in Table 3.3 demonstrate that adding VIX futures has a beneficial effect on portfolio performance, improving mean return but most importantly reducing the volatility. Comparing the performance of the six portfolios under investigation it is also clear that, in normal times such as the period 2004-2007 adding VIX futures contract improves the mean return and produces an excellent Sharpe ratio and of course improves VaR risk measures[7]. Moreover, during turbulent times such as 2008-2012, there is a great benefit in having VIX futures in the investment portfolio, the mean return staying positive and Sharpe ratio being the best for the portfolios containing VIX futures positions. Looking at the event risk of 2008 it can also be remarked that extreme losses can be avoided if VIX futures positions are added.

	SPX	**97.5% SPX 2.5% VIX Futures**	**90% SPX 10% VIX Futures**	**60% SPX 40% Bonds**	**58.5% SPX 39% Bonds 2.5% VIX Futures**	**54 % SPX 36% Bonds 10% VIX Futures**
All sample (2004- 2012)						
Annualized Mean return	5.11%	5.50%	6.76%	4.91%	5.36%	6.78%
Volatility	22.25%	20.35%	15.80%	12.92%	11.34%	8.90%
Min	-9.03%	-8.43%	-6.90%	-5.46%	-4.86%	-3.60%
Max	11.58%	10.75%	8.39%	6.72%	6.10%	4.30%
Skew	-0.0390	0.0000	0.2670	-0.1179	-0.0409	0.8430
Excess Kurtosis	9.9619	10.7259	12.3318	9.9720	11.3121	11.3938
Annual Sharpe ratio	17.05%	20.53%	34.44%	27.77%	35.58%	61.32%
VaR 1%(historical)	4.43%	4.04%	2.91%	2.50%	2.19%	1.55%

[6] In Tables x-y from the Appendix we investigate the robustness of our results to changing the assumptions. For example, in Table x we report the results obtained assuming daily rather than rebalancing. Moreover, results reported in Tables 3.3 and 3.4 assume that the notional amount of the futures is held in cash. An alternative would be to invest this amount in the risk free asset and post this as margin. We refer to this case as the 'collateralized futures' case. We examine the impact of collateralization in Tables x and xx from the Appendix. We note that whether or not we take into consideration the collateralization for marking to market the futures contracts, does not have an impact on our final conclusions.

[7] Interestingly, when using daily rebalancing as shown in the appendix, during this period adding only 2.5% VIX futures leads to a better performance than when adding 10% VIX futures.

VaR 5% (historical)	2.13%	1.94%	1.37%	1.22%	1.03%	0.64%
subperiod 1: 2004 - 2007						
Annualized Mean return	8.30%	8.70%	9.91%	6.57%	7.02%	8.38%
Volatility	12.09%	10.84%	8.94%	7.22%	6.22%	6.52%
Min	-3.47%	-2.64%	-1.90%	-1.88%	-1.40%	-1.59%
Max	2.92%	2.59%	3.64%	1.85%	1.41%	3.79%
Skew	-0.2767	-0.2321	0.7687	-0.2049	-0.1105	2.3308
XS Kurt	1.9129	1.5462	3.7727	1.4816	0.8776	14.8792
Annual Sharpe ratio	48.37%	57.59%	83.47%	57.04%	73.40%	90.84%
VaR 1%(historical)	2.22%	2.00%	1.37%	1.22%	1.00%	0.77%
VaR 5% (historical)	1.27%	1.10%	0.78%	0.76%	0.65%	0.49%
subperiod 2: 2008-2012						
Annualized Mean return	2.21%	2.59%	3.90%	3.39%	3.85%	5.32%
Volatility	28.50%	26.15%	20.10%	16.47%	14.51%	10.61%
Min	-9.03%	-8.43%	-6.90%	-5.46%	-4.86%	-3.60%
Max	11.58%	10.75%	8.39%	6.72%	6.10%	4.30%
Skew	-0.0005	0.0324	0.2081	-0.0790	-0.0134	0.4754
XS Kurt	6.0670	6.5080	7.9647	6.2421	7.1060	8.3145
Annual Sharpe ratio	6.75%	8.81%	17.95%	18.85%	24.52%	47.45%
VaR 1%(historical)	5.24%	4.79%	3.60%	2.98%	2.68%	1.77%
VaR 5% (historical)	2.90%	2.62%	1.88%	1.61%	1.35%	0.94%
short crisis subperiod: 2008						
Annualized Mean return	-42.23%	-39.74%	-31.95%	-24.35%	-22.07%	-14.99%
Volatility	41.37%	38.43%	30.36%	24.03%	21.61%	15.75%
Min	-9.03%	-8.43%	-6.90%	-5.46%	-4.86%	-3.60%
Max	11.58%	10.75%	8.39%	6.72%	6.10%	4.30%
Skew	0.1999	0.2116	0.2942	0.0925	0.1310	0.3818
XS Kurt	3.8773	4.0208	4.4947	3.8998	4.1829	4.9141
Annual Sharpe ratio	-104.37%	-105.89%	-108.39%	-105.29%	-106.49%	-101.22%
VaR 1%(historical)	8.24%	7.66%	6.22%	4.97%	4.47%	3.20%
VaR 5% (historical)	4.52%	4.11%	2.90%	2.59%	2.23%	1.45%

Tables 3.3: Performance of volatility-diversified US portfolios

Notes: *The performance statistics are of the daily relative returns on the different portfolios. The portfolios are weekly rebalanced, and the notional of the futures contracts is assumed to be held in cash (no collateralization of the futures).*

	STOXX	97.5% STOXX 2.5% VSTOXX Futures	90% STOXX 10% VSTOXX Futures	60% STOXX 40% Bonds	58.5% STOXX 39% Bonds 2.5% VSTOXX Futures	54 % STOXX 36% Bonds 10% VSTOXX Futures
Annualized Mean return	5.42%	5.97%	7.68%	5.02%	5.58%	**7.31%**
Volatility	25.51%	23.42%	17.92%	15.05%	13.28%	**9.51%**
Min	-6.33%	-5.81%	-5.21%	-3.77%	-3.22%	-3.10%
Max	10.35%	9.43%	6.78%	6.48%	5.72%	3.54%
Skewness	0.1616	0.1765	0.1912	0.2552	0.3047	0.4004
Excess Kurtosis	3.3844	3.3979	3.4526	3.9618	4.1059	3.9719
Annualized Sharpe ratio	10.66%	13.98%	27.82%	15.45%	21.73%	**48.45%**
VaR 1%(historical)	4.28%	3.83%	2.69%	2.65%	2.15%	**1.42%**
VaR 5% (historical)	2.55%	2.32%	1.78%	1.57%	1.36%	**0.88%**

Tables 3.4: Performance of volatility-diversified European portfolios

Notes: ***The performance statistics are of the daily relative returns on the different portfolios. The portfolios are weekly rebalancing, and the notional of the futures contracts is assumed to be held in cash (no collateralization of the futures).***

A similar story follows from the results of Table 3.4, although this analysis covers only most recent period due to the availability of VSTOXX futures contracts introduced by EUREX.

For the European case, our analysis shows that, for the period under analysis (i.e. May 2009 – February 2012), adding volatility exposure to an equity portfolio that tracks the EURO STOXX 50 provides indeed risk diversification benefits: the volatility decreases from over 25% to under 18% (i.e. a reduction of around 30%) for a 10% exposure to VSTOXX futures (nearest maturity). Downside risk, as measured by Value-at-Risk, computed using the historical methodology for two different significance levels, 1% and 5%, also decreases. Moreover, the average return also increases, from a (annualized daily) value of 5.42% to 7.68% (an increase of 40%), resulting in a very significant increase in the annualized Sharpe ratio, from less than 0.06 to over 0.21, an almost 4-fold increase. A reduction in volatility coupled with an increase in returns is also obtained by investing as little of 2.5% of the portfolio value in VSTOXX futures, only that improvements are more moderate in this case.

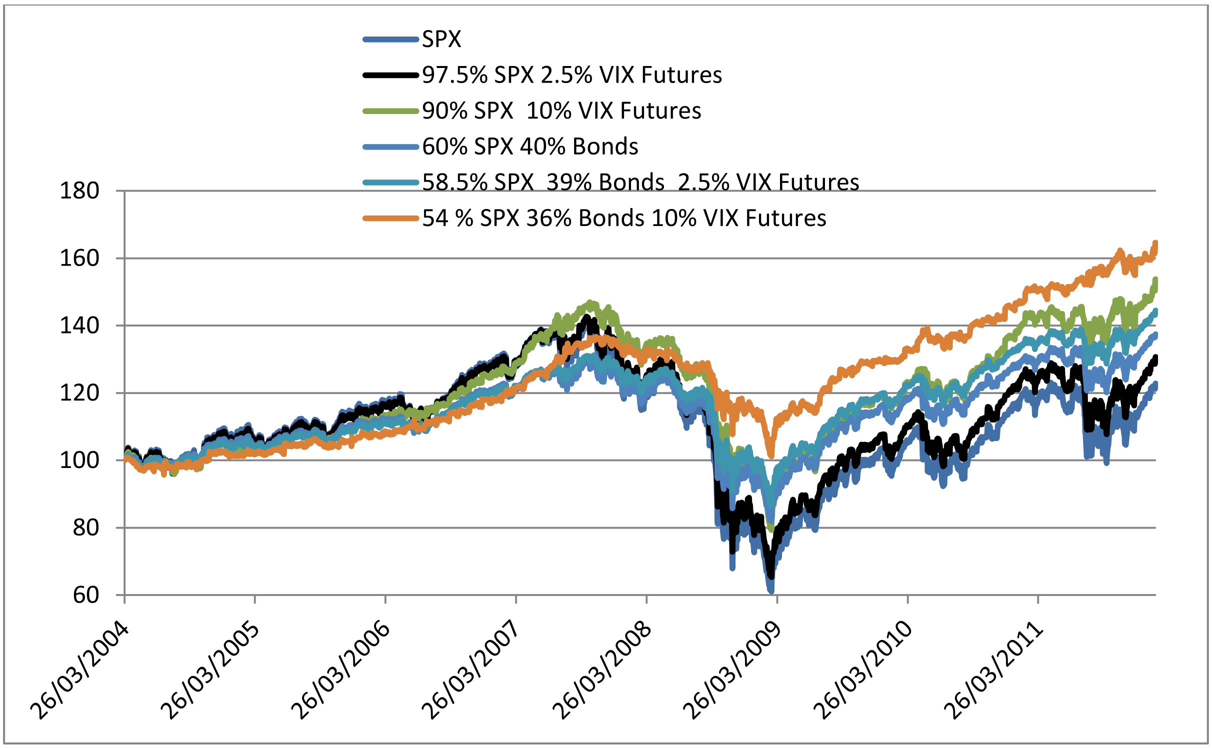

Fig. 3.7 Comparative Performance of various portfolios based on S&P 500

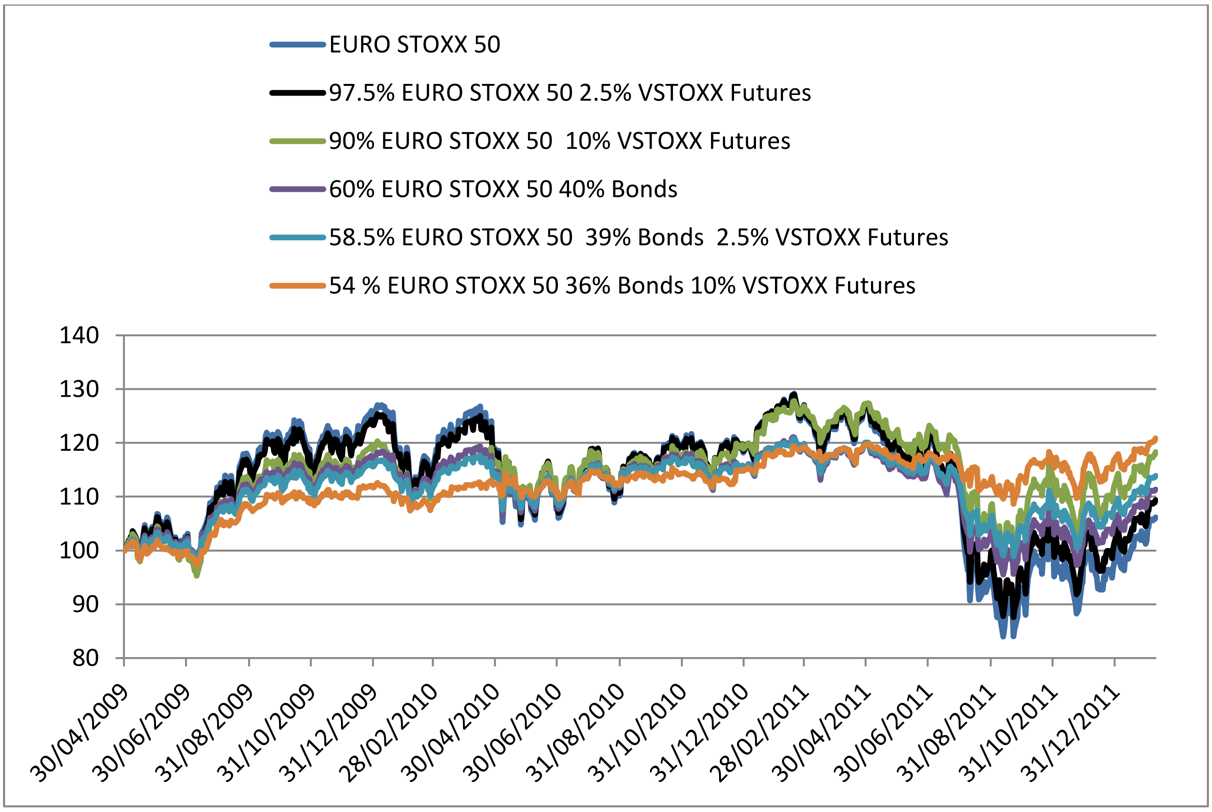

Fig. 3.8 Comparative Performance of various portfolios based on EURO STOXX 50

In Figures 3.7 and 3.8 we have compared various portfolios combining equity positions, bond positions and volatility index positions. Overall it can be seen that VSTOXX and VIX futures contracts can help investors to preserve positive returns after unexpected shocks in the equity markets. On the other hand, over periods of market calmness, the futures contracts are more of a break, confirming similar analyses in Szado (2009) and Rhoads (2011).

4. Modelling the VIX-VSTOXX difference

In this section we investigate the nature of the difference between the VIX and VSTOXX volatility indices. If significant, we seek to exploit this difference in a trading strategy, hence we work with futures prices on the two volatility indices rather than with their respective spot levels. Since these are the most actively traded contracts, we employ the nearest maturity futures contracts both for the VIX as well as for the VSTOXX. We start by testing whether this difference is statistically significant and we then proceed to modelling the stochastic behaviour of the difference by means of discrete-time GARCH modelling.

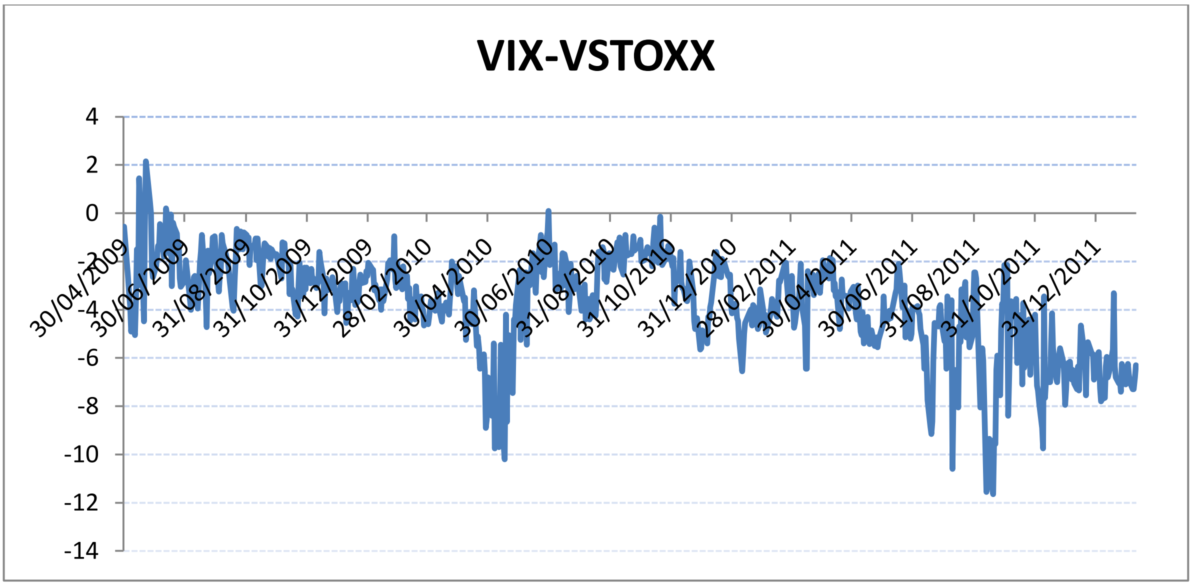

Figure 4.1 VIX-VSTOXX Futures Historical Difference

Figure 4.1 plots the daily series of differences between the VIX and the VSTOXX nearest maturity futures prices, for a period of over 3 years, ranging from 30th April 2009 (when the futures contracts on VSTOXX were first introduced) to the 9th February 2012, while Table 4.1 summarizes the main statistics for this series. From Figure 4.1, we can infer that the VIX-

VSTOXX futures prices difference series appears to be stationary and also characterized by ARCH effects. Both features are confirmed by the ADF and ARCH test results, respectively (see Table 4.1) which are significant even at the 1% level.

Mean	**-3.7769*** **
t stat mean	-48.3589
Std dev	2.0649
Min	-11.65
Max	2.15
Skewness	-0.8444***
t stat skew	-9.1145
Excess Kurtosis	0.8628***
t stat kurt	4.6562
ARCH test	273.59***
ADF test	-4.168378***

Table 4.1: Summary Statistics for the VIX-VSTOXX Futures Difference

Notes: *The summary statistics are of the difference between the VIX and VSTOXX nearest maturity futures prices, from 30th April 2009 to 9th February 2012. Asterisks denote significance at 10% (*), 5% (**) and 1%(***). The standard error of the sample mean is equal to the sample standard deviation, divided by the square root of the sample size, while the standard errors are approximately* $(6/T)^{1/2}$ *and* $(24/T)^{1/2}$ *for the sample skewness and excess kurtosis, respectively, where T is the sample size.*

The difference between the nearest futures prices of the two volatility indices appears significant and negative, which means that the volatility implied by the EURO STOXX 50 options was significantly (expected to be) higher than that of S&P 500 options, at least for the period under consideration. The series also exhibits non-normality features in the higher moments – namely significant negative skewness and significant positive kurtosis – further advocating the use of GARCH modelling which can (at least partially) also explain these features.

Below we shall estimate a number of models from the GARCH family in order to see which one best captures the dynamics of the difference series; furthermore, as models from the GARCH family also lend themselves to forecasting applications, we shall also consider the forecasts implied by these models. A very brief description of this family of models follows.

Engle's (1982) seminal paper introduced the class of autoregressive conditional heteroskedastic (ARCH) models, which Bollerslev (1986) generalized into GARCH. Any model pertaining to this class of models is essentially formed of two equations:

- A conditional mean equation, which is a regression model describing the evolution of the financial series under analysis;
- A conditional variance equation, which describes the conditional variance dynamics;

A very general specification of a GARCH model is given by:

$$
\begin{aligned}
& y_t = E(y_t \mid \Omega_{t-1}) + \varepsilon_t \\
& \varepsilon_t = z_t \sigma_t \\
& z_t \sim D(0,1) \\
& \sigma_t = f(\{\varepsilon_{t-i}\}, \{\sigma_{t-j}\}, \{X_{t-1}\} \forall i \geq 1, j \geq 1)
\end{aligned}
\tag{3}
$$

In the above set of equations, y_t denotes financial time series under analysis, in our case this will be the difference series described above; $E(y_t \mid \Omega_{t-1})$ denotes the conditional mean of this difference, while ε_t is a disturbance process. $\{z_t\}$ is a sequence of *i.i.d* random variables with (zero mean and unit variance) probability distribution *D*. The last equation provides an expression for the conditional standard deviation; X_t is a vector of predetermined variables included in the information set Ω_t, available at time *t*.

A plethora of models have been developed in the literature following Engle and Bollerslev's seminal papers, many of them listed in a recent and very useful glossary compiled by Bollerselv (2008). In order to find the most appropriate GARCH model to explain the VIX-VSTOXX difference (which was shown above to have ARCH effects), we first focus on the specification of the mean equation; once we arrived at an optimal model for the mean equation we consider alternative error distributions and conditional variance specifications to see which yields the best forecasts of the difference.

We start from the plot of the autocorrelation and partial autocorrelation functions of the difference series (see Figures 4.2 and 4.3). These figures reveal a gradually decaying ACF and a PACF which decays to zero much faster, taking significantly non-zero values for the first few lags and then becoming insignificant, with the exception of very few lags.

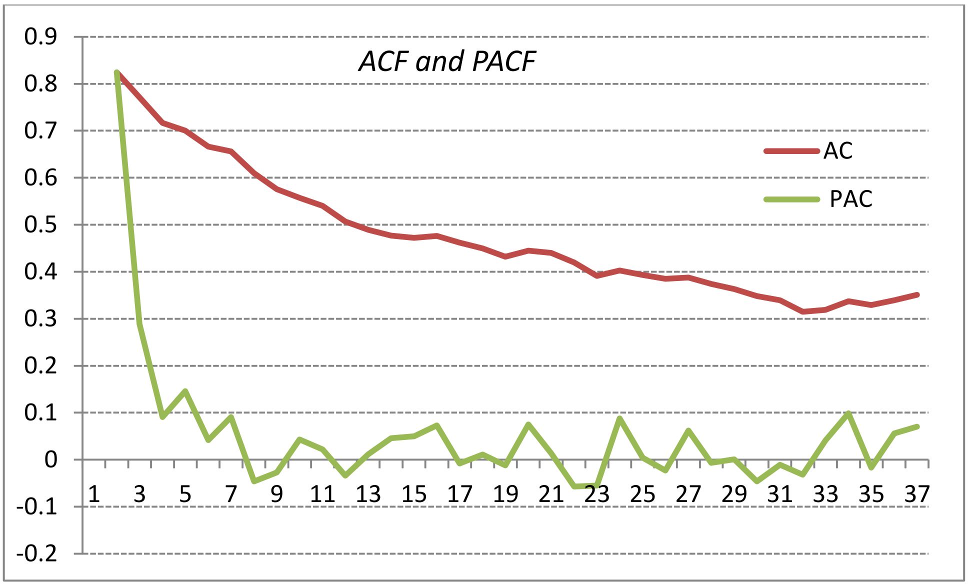

Figure 4.2 ACF and PACF for the VIX-VSTOXX Nearest Futures Difference

Autocorrelation	Partial Correlation	Lag
.|******|	.|******|	1
.|******|	.|** |	2
.|***** |	.|* |	3
.|***** |	.|* |	4
.|***** |	.|. |	5
.|***** |	.|* |	6
.|**** |	.|. |	7
…	…..	….
.|*** |	.|. |	17
.|*** |	.|. |	18
.|*** |	.|* |	19
.|*** |	.|. |	20
.|*** |	.|. |	21
.|*** |	.|. |	22
.|*** |	.|* |	23
.|*** |	.|. |	24
…	…	…
.|** |	.|. |	32
.|** |	.|* |	33
.|** |	.|. |	34
.|** |	.|. |	35
.|** |	.|. |	36

Figure 4.3 Significance of the ACs and PACs for the VIX-VSTOXX Nearest Futures Difference

A model from the ARMA family should be able to account for the autocorrelation in the series. Indeed the results in Table 4.2 show that a constrained AR(4) model (with the coefficient on the third lag constrained to be equal to zero) is the most parsimonious model that eliminates the autocorrelation. It also minimizes the BIC criterion, all terms included in the regression (namely the AR(1), AR(2) and AR(4) terms) are significant and the improvements in the other information criteria – AIC, HQIC – as well as the log likelihood are only minimal for some of the competing models from Table 4.2. We therefore proceed to GARCH estimation, based on a constrained AR(4) mean equation.

Criteria\Model	AR(1)	AR(2)	AR(3)	AR(4)	**AR(4) constrain AR(3)=0**	ARMA (1,1)	ARMA (2,1)	ARMA (2,2)
AIC	3.1435	3.0564	3.0502	3.0313	**3.0285**	3.0348	3.0279	3.0293
BIC	3.1566	3.0760	3.0764	3.0640	**3.0547**	3.0543	3.0540	3.0620
HQIC	3.146	3.0640	3.0603	3.0439	**3.0387**	3.0423	3.0380	3.0420
Log likelihood	-1095.096	-1062.15	-1057.48	-1048.37	**-1048.41**	-1056.13	-1051.23	-1050.72
AR(1) signif	***	***	***	***	*******	***	***	***
AR(2) signif	-	***	***	***	*******	-	**	NO
AR(3) signif	-	-	**	NO	**-**	-	-	-
AR(4) signif	-	-	-	***	*******	-	-	-
MA(1) signif	-	-	-	-	**-**	***	***	**
MA(2) signif	-	-	-	-	**-**	-	-	NO
Ljung-Box	Autocorr at lag 1	No autocorr at lag 1, but lag 2 signif	No autocorr up to lag 2, but signif at 3	No autocorr	No autocorr	No autocorr at 1% signif.	No autocorr at 1% signif.	No autocorr at 1% signif.

Table 4.2 ARMA model selection

Notes: AIC, BIC, HQIC stand for the Akaike, Bayesian and Hannan-Quinn information criteria. The optimal model, according to a particular information criterion, should minimize the respective information criterion. The log likelihood should be maximized by the optimal model. Asterisks denote significance at 10% (*), 5% (**) and 1%(***).

The GARCH model in (3) now becomes:

$$\begin{aligned} y_t &= c_0 + c_1 y_{t-1} + c_2 y_{t-2} + c_4 y_{t-4} + \varepsilon_t \\ \varepsilon_t &= z_t \sigma_t \\ z_t &\sim D(0,1) \\ \sigma_t &= f(\{\varepsilon_{t-i}\}, \{\sigma_{t-j}\}, \{X_{t-1}\} \forall i \geq 1, j \geq 1) \end{aligned} \quad (4)$$

where the error distribution D will be either the normal or the (standardized) Student-t.

We now turn our attention to the final equation in (4), the conditional variance equation, where the focus of a GARCH model lies. Three different variance specifications are considered in this paper: the classical symmetric GARCH (1, 1) of Bollerslev (1986) and two asymmetric specifications, the exponential GARCH (EGARCH) model of Nelson (1991) and the GJR model, first introduced by Glosten, Jagannathan and Runkle (1993). The choice of these particular three versions out of the great variety of GARCH models available is not random. The basic GARCH (1, 1) model offers the advantage of having a simple specification of the conditional variance equation. This is especially important in a forecasting exercise. Even if more elaborate models tend to fit better in sample, parsimonious models are preferred in prediction because they have more degrees of freedom. Moreover, previous empirical studies have proved that no more than a GARCH (1, 1) is needed to account for volatility clustering.[8] However, in equity markets, volatility tends to increase more following unexpectedly large negative returns than following unexpected positive returns of the same magnitude. To capture this asymmetry in volatility, often attributed to the "leverage effect" (i.e. a fall in the market value of a firm will increase its degree of leverage), more than a GARCH (1, 1) is needed. Both the GJR and the EGARCH models allow for asymmetric responses of volatility to positive and negative shocks respectively. Hence, the final equation in (2) will, in turn, take one of the following forms:

$$GARCH(1,1): \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$
$$GJR: \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \lambda\varepsilon_{t-1}^2 1(\varepsilon_{t-1} < 0)$$
$$EGARCH: \ln(\sigma_t^2) = \omega + \alpha\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - E\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}}\right]\right] + \beta\ln(\sigma_{t-1}^2) + \lambda\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} \quad (5)$$

where $1(\varepsilon_t < 0) = \begin{cases} 1, \text{ if } \varepsilon_t < 0 \\ 0, \text{ otherwise} \end{cases}$.

Since the variance is always a positive quantity, non-negativity constraints apply for GARCH(1,1) and GJR: in both models $\omega>0$, α, $\beta \geq 0$; for the latter model, $\alpha + \gamma \geq 0$ is also sufficient for non-negativity.[9] One advantage of the EGARCH model is that it does not necessitate any non-negativity constraints; Moreover, for the leverage effect to hold we would need $\gamma>0$ for the GJR and $\gamma<0$ for the EGARCH. The coefficients of the GARCH models are estimated using the

[8] For example, Berkowitz and O'Brien (2002) show that VaR forecasts based on a simple ARMA(1, 1)-GARCH(1,1) model were at least as accurate as those produced by the complicated structural models employed by six large commercial banks.

[9] Parameter conditions that ensure that the conditional variance converges to a finite unconditional variance are given in Table x from the appendices. We note that, for all 6 models considered, the parameter estimates reported in Table 4.3 satisfy these convergence conditions.

technique of Maximum Likelihood (ML).[10] In the interest of clarity, the full details of the estimated GARCH models are summarized in Appendix D, while the estimation results obtained for alternative GARCH models are reported in Table 4.3 below.

Model	AR(4)-N-GARCH(1,1)	AR(4)-T-GARCH(1,1)	AR(4)-N-GJR	AR(4)-T-GJR	AR(4)-N-EGARCH	AR(4)-T-EGARCH
Constant	-0.2951***	-0.2581***	-0.3381***	-0.2934***	-0.3011***	-0.2819***
AR(1)	0.5837***	0.6163***	0.5758***	0.6084***	0.5835***	0.6083***
AR(2)	0.1636***	0.1627***	0.1762***	0.1711***	0.1766***	0.1712***
AR(4)	0.1701***	0.1456***	0.1562***	0.1369***	0.1599***	0.1414***
ω	0.0251***	0.0323**	0.0193***	0.0232**	-0.2042***	-0.1811***
α	0.1381***	0.1142***	0.0481**	0.0341	0.2586***	0.2277***
β	0.8437***	0.8547***	0.8910***	0.9014***	0.9753***	0.9681***
λ	-	-	0.0805**	0.0715*	-0.0325	-0.0381
df	-	7.2035***	-	7.4292***	-	7.7875***
Log Likelihood	-947.651	-936.259	-946.426	-935.283	-944.298	-934.196

Table 4.3 GARCH Model Estimation

Note: Asterisks denote significance at 10% (*), 5% (**) and 1%(***).

The results in Table 4.3 show that all GARCH models considered fit very well in sample. For the symmetric models (i.e. the normal and Student-t GARCH(1,1) models) all the estimated parameters are highly significant. Among the asymmetric specifications considered, only for the normal GJR all the model parameters are significant. Although not reported in this table because of lack of space, we also estimated GARCH-in-mean versions for all the models in Table 4.3 (i.e. we added an additional regressor to the conditional mean equation, which was either the conditional variance, or its square root or its natural logarithm). However, the GARCH-in-mean

[10] Note that for the EGARCH models we actually estimated slightly restricted versions of the specification given in (2), namely: $\ln(\sigma_t^2) = \omega_0 + \alpha\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}}\right] + \beta\ln(\sigma_{t-1}^2) + \lambda\frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}$; this restriction however has no impact on the parameter estimates α, β and λ and $\omega_0 = \omega + \alpha E\left[\frac{|\varepsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}}\right]$.

terms were insignificant for all 18 specifications that we estimated and hence results are not reported here.

5. Investment Strategies Based on Our Results

Knowing that the difference between the VSTOXX and VIX is significant we investigate first the following trading strategy. We enter into a cross-country spread, long VSTOXX futures and short VIX futures when the difference of the settlement prices for the two contracts is larger than 3% and we unwind the first day this difference becomes less than 1%. In Figure 5.1 we report the cumulative returns for each leg of the strategy. The profit that could have been made is in EUR for the VSTOXX curve and in USD for the VIX curve.

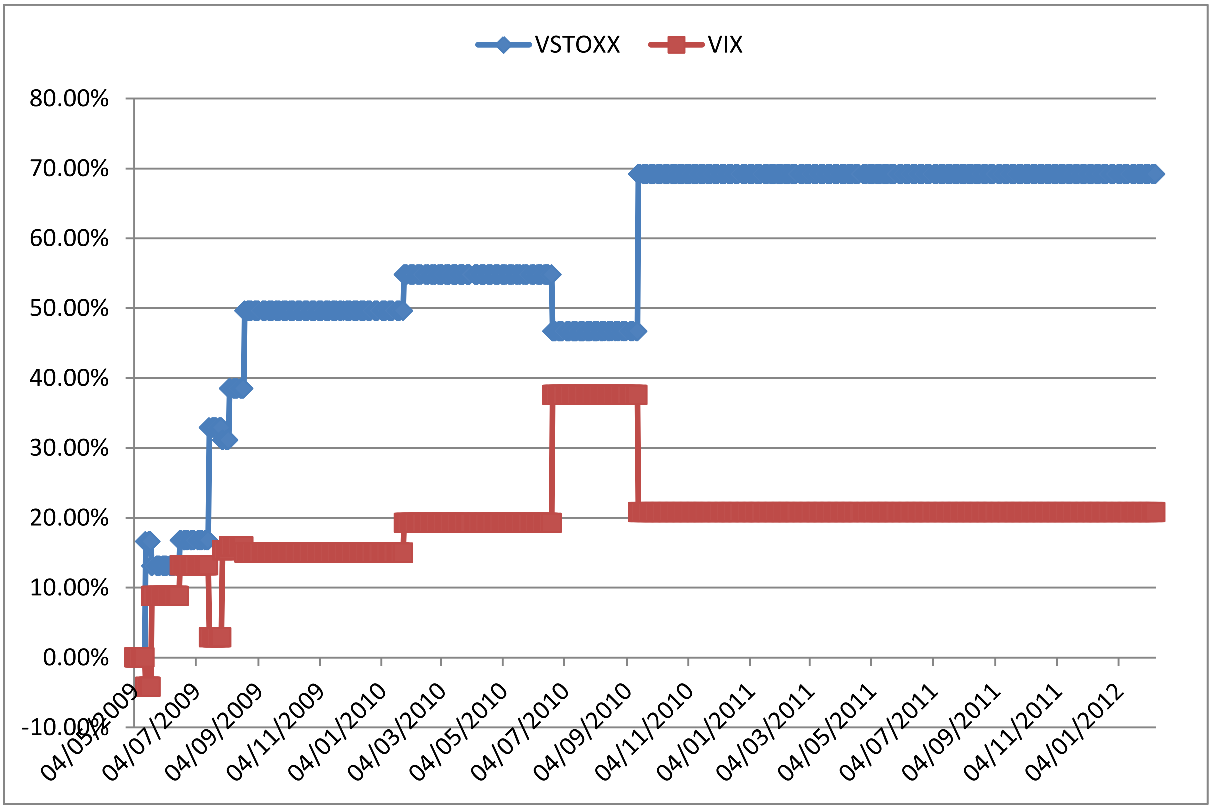

Figure 5.1 Cumulative returns from long-short trading strategy using VSTOXX futures and VIX futures with nearest maturity. Calculations are for the period 30 April 2009 to 9th February 2012.

The trading strategy highlighted above is more profitable for the VSTOXX leg than the VIX leg. One explanation is given by the fact that in 2010 and 2011 the European sovereign financial crisis led to a higher level of VSTOXX and VSTOXX derivatives in general.

A potential application of the GARCH modelling results is for the forecasting of the VIX-VSTOXX (nearest futures price) difference which in turn can be used to inform trading strategies. Figure 5.2 plots the series of one-step ahead forecasts obtained from a AR(4)-Normal-GJR (see Appendix D, Table D.1 for the exact model specification and Table 4.3, Column 4 for the estimation results: this is the best fitting model which also exhibits asymmetry). The model parameters are re-estimated daily, using a rolling sample of 500 observations, with 199 observations used for out-of-sample forecasting. The results depicted in Figure 5.2 show that the VIX-VSTOXX Futures difference remains negative for the entire forecasting period (i.e. April 2011-February 2012). This is not surprising given that during this period the European markets have been affected by the recent European sovereign debt crisis, which had a much lesser impact on the US market. We also note that our model correctly forecasts the sign of the difference throughout the observation period.

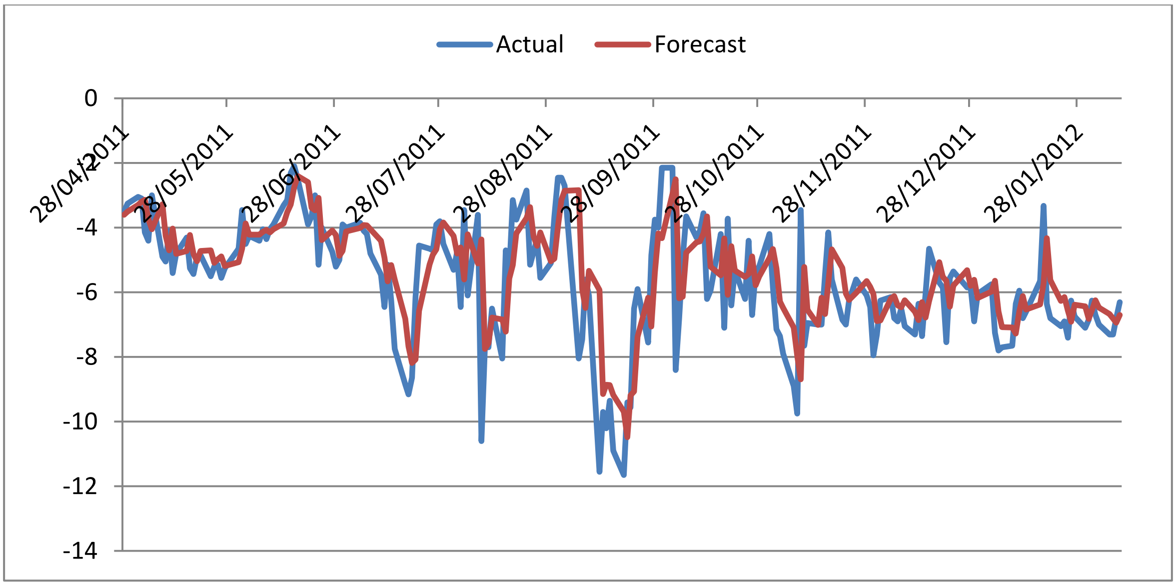

Figure 5.2 One-step ahead forecasts of the VIX-VSTOXX nearest Futures Price Difference

Given that the VIX-VSTOXX futures price difference is negative throughout our forecasting evaluation period (and has a significant negative mean throughout the entire sample), we now compare the trading profit obtained for the following long-short strategies:[11]

1) Long the nearest maturity (M1) VSTOXX futures and short the nearest maturity VIX futures
2) (Dynamic long-short strategy): We start the strategy long the nearest maturity VSTOXX futures and short the nearest maturity VIX futures the first time our AR(4)-N-GJR model forecast an increase of the spread in absolute value and unwind when the model signals a reduction in spread.
3) A second dynamic strategy is given by a signal to trade the spread, long VSTOXX and short VIX, when the difference between the daily spread forecast and the current spread is greater than a given threshold. The positions are closed at the end of each day.

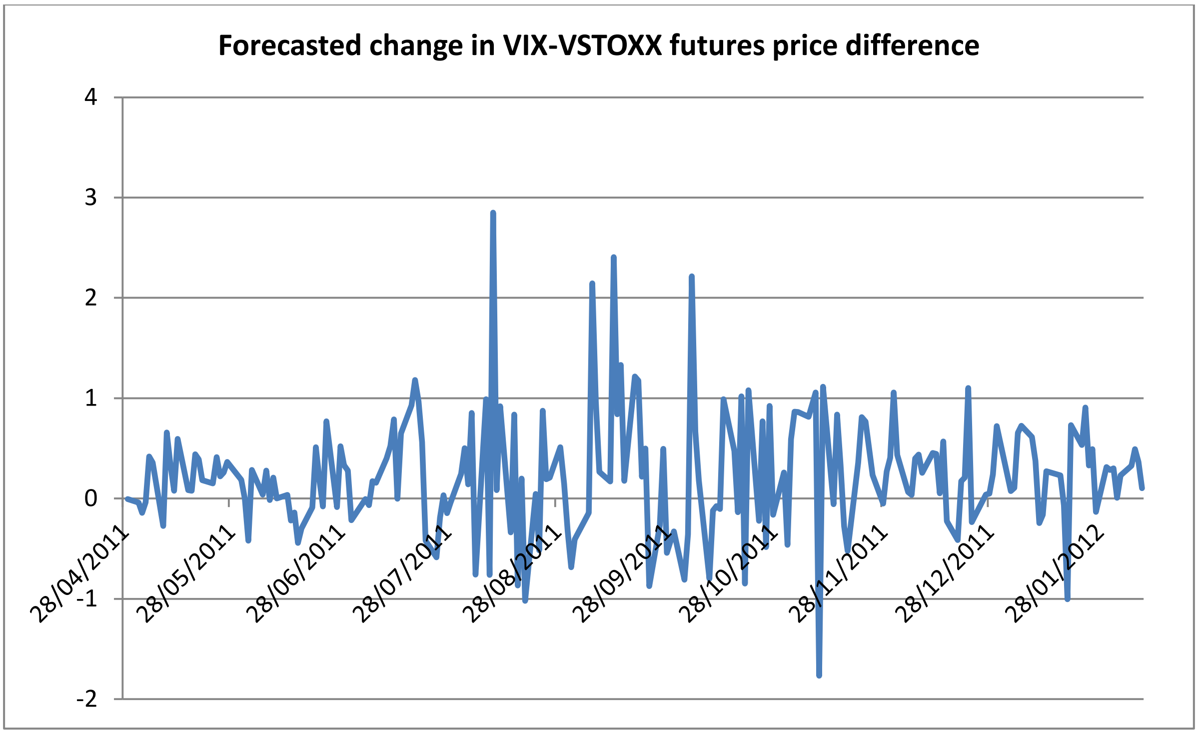

Figure 5.3 Forecasted change in the VIX-VSTOXX nearest futures price difference

[11] We ignore for the moment any FX risk or indivisibility of the futures contracts and assume that an investor has the same exposure to both the VIX and VSTOXX, through their respective futures contracts.

Note: Since the difference is negative throughout, a positive change will signify a decrease in the VIX-VSTOXX nearest futures price difference.

For the first strategy the cumulative log-return for the VSTOXX leg was 26.14% and for the VIX leg was -17.73%.

The performance of the second trading strategy is illustrated in Figure 5.4. The trading leg associated with VIX provides excellent return, offsetting the performance of the VSTOXX leg.

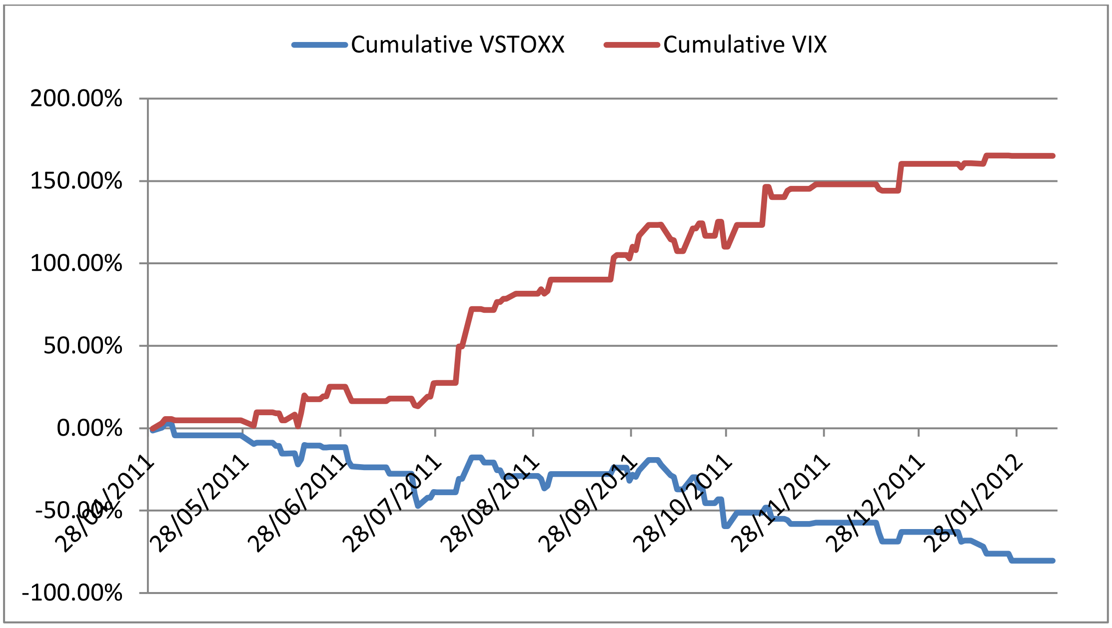

Figure 5.4 Performance of dynamic trading strategy. Cumulative log-returns are calculated for each leg of the trading strategy.

Note. Calculations are done for the period 28 April 2011 to 9 February 2012.

The graph in Figure 5.5 displays the performance of our second dynamic strategy with a threshold equal to 0.5. This strategy seems to work much better, taking advantage of the excellent forecast of the spread.

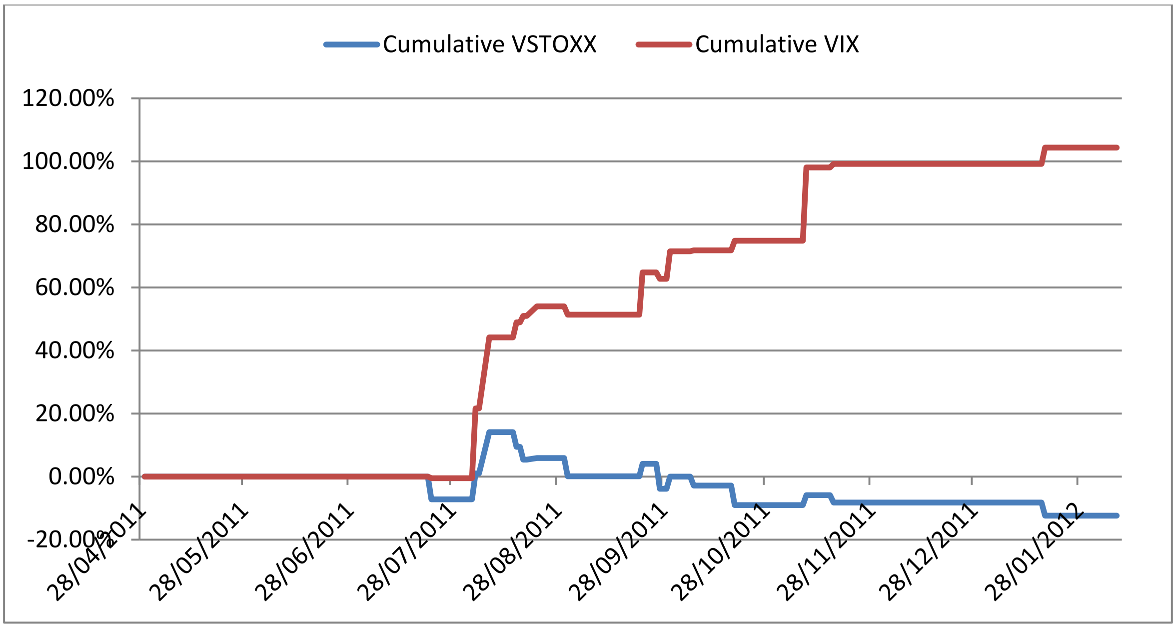

Figure 5.5 Performance of the second dynamic trading strategy. Every day when the difference between the forecast spread and the current spread is greater than 0.5, a long position in VSTOXX Futures and short position of VIX Futures is taken. These positions are closed at the end of the day. Cumulative log-returns are calculated for each leg of the trading strategy.

Note. Calculations are done for the period 28 April 2011 to 9 February 2012.

6. Conclusions

The negative correlation between VSTOXX and EURO STOXX 50 is quite stationary and it fluctuates mostly between -50% and -95%. The evolution of the correlation between VIX and S&P500 was mixed. There is also a clear discrepancy between the correlation between S&P 500 and VIX on one side and the correlation between the S&P 500 and the VIX futures with nearest maturity. A similar conclusion can be drawn for STOXX. Moreover, it seems that the futures with the second maturity produces a closer resemblance to the VIX (VSTOXX).

We confirm on an extended set of data for VIX and also on a new set of data for VSTOXX that these volatility indexes predicted correctly that the contemporaneous realized high volatilities observed in the market after market shocks such as Lehman collapse and the euro crisis in Europe, were unsustainable and the equity markets will calm down after a short period of time.

The first major contribution of the paper is to use the methodology described in Szado (2009) and demonstrate that using VIX and VSTOXX futures improves the return-risk profile of investment portfolios, particularly during turbulent times. The benefits seem to be larger for VSTOXX, although there is less historical data involving futures contracts.

The second major contribution of this paper is to tackle the data for U.S. and Europe with a battery of state-of-the art GARCH models. Identifying a GARCH model that works well with data allows investors to engage in directional trading given by the signal produced by the GARCH model. We have identified three models that work well, the GARCH (1,1) widely known and applied in the literature, the EGARCH and the GJR models that are capable to capture the asymmetry behind the leverage effect in equity markets. We have shown how the AR(4)-N-GJR model can be employed successfully to trade cross-border volatility futures.

References

Ahoniemi, K. (2008). Modeling and Forecasting the VIX Index. Working paper, Helsinki School of Economics and HECER.

Banerjee, P.S., Doran, J.S. & Peterson, D.R. (2007). Implied Volatility and Future Portfolio Returns. *Journal of Banking & Finance*, 31, 3183-3199.

Becker, R., Clements, A.E. & White, S.I. (2006). On the Informational Efficiency of S&P500 Implied Volatility. *North American Journal of Economics and Finance*, 17, 139-153.

Berkowitz, J. & O'Brien, J. (2002) How Accurate are Value-at-Risk Models at Commercial Banks? *The Journal of Finance* 57:3, 1093-1111.

Blair, B.J., Poon, S-H. & Taylor, S.J. (2001). Forecasting S&P 100 Volatility: the Incremental Information Content of Implied Volatilities and High-frequency Index Returns. *Journal of Econometrics*, 105, 5-26.

Bollerslev, T. (1986) Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics*, 31, 307-327.

Bollerslev, T. (2008). Glossary to arch (garch). Technical report, Glossary to ARCH (GARCH). Research Paper 2008-49.

Brenner, M., Shu, J. & Zhang, J.E. (2007) The Market for Volatility Trading; VIX Futures, working paper/

Brockhaus, O. & Long, D. (2000).Volatility swaps made simple, *Risk* 19(1), 92–95.

Campbell, J. Y., & Shiller J. R. (1998) Valuation Ratios and the Long-Run Stock Market Outlook. *The Journal of Portfolio Management*, 24, pp. 11-26.

Cipollini, A. & Manzini, A.(2007) Can the VIX Signal Market Direction? An Asymmetric Dynamic Strategy., working paper http://ssrn.com/abstract=996384

Chen, Hsuan-Chi, Chung, San-Lin, & Ho, Keng-Yu. (2011). The diversification Effects of Volatility-Related Assets. *Journal of Banking and Finance*, 35, 1179-1189.

Christoffersen, P.F. & Diebold, F.X. (2006). Financial Asset Returns, Direction-of-Change Forecasting, and Volatility Dynamics. *Management Science*, 52, 1273-1287.

Corrado, C.J., & Miller, T.W., (2005). The Forecast Quality of CBOE Implied Volatility Indexes. *The Journal of Futures Markets*, 25, 339-373.

Dash, S., & Moran, M.. (2005). VIX as a companion for hedge fund portfolios. *The Journal of Alternative Investments*, 8 (3), 75–80.

Engle, R.F. (1982) Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 50 (4), 987-1007.

Fernandes, M., Medeiros, M.C. & Scharth, M. (2007). Modeling and Predicting the CBOE Market Volatility Index. Working paper, Pontical Catholic University of Rio de Janeiro.

Giot, P. (2005). Relationships Between Implied Volatility Indexes and Stock Index Returns. *Journal of Portfolio Management*, 31, 92-100.

Glosten, L.R., R. Jagannathan & Runkle, D.E. (1993) On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *Journal of Finance*, 48 (5), 1779-1801.

Jiang, G.J. & Tian, Y.S. (2007). Extracting Model-Free Volatility from Option Prices: An Examination of the VIX Index. *The Journal of Derivatives*, 14(3), 35-60.

Konstantinidi, E., Skiadopoulos, G. & Tzagkaraki, E. (2008). Can the Evolution of Implied Volatility be Forecasted? Evidence from European and US Implied Volatility Indices. *Journal of Banking & Finance*.

Lin, Y. (2007), Pricing VIX futures: Evidence from integrated physical and risk-neutral probability measures, *Journal of Futures Markets*, 27(12), 1175.

Moran, M., & Dash, S (2007).VIX Futures and Options. *The Journal of Trading*, 2 (3), 96–105.

Nelson, D.B. (1991) Conditional Heteroskedasticity in Asset Returns: A New Approach *Econometrica*, 59 (2), 347-370.

Rhoads, R. (2011). *Trading VIX Derivatives: Trading and Hedging Strategies Using VIX Futures, Options, and Exchange Traded Notes*, Wiley, Chichester.

Simon, D.P. (2003). The Nasdaq Volatility Index During and After the Bubble. *Journal of Derivatives*, 11(2), 9-22.

Szado, E. (2009) VIX Futures and Options: A Case Study of Portfolio Diversification During the 2008 Financial Crisis. *The Journal of Alternative Investments*, 12 , 68–85.

Whaley, R. (1993). Derivatives on market volatility: Hedging tools long overdue. *Journal of Derivatives*, 1, 71–84.

Whaley, R. (2000). The Investor Fear Gauge. *The Journal of Portfolio Management,* 26 (2000), 12-17.

Whaley, R. (2009). Understanding the VIX. *Journal of Futures Markets*, Spring, 98–105.

Zhang, J. & Zhu, Y. (2006), VIX Futures, *Journal of Futures Markets*, 26(5), 521–531.

Appendices

Appendix A Descriptive Statistics

Summary Statistics of VIX (02/01/1990 until 01/03/2012 daily)

	VIX CLOSE	VIX HIGH	VIX LOW	VIX OPEN
Mean	20.557	21.382	19.882	20.605
Standard Deviation	8.249	9.024	8.096	8.615
Skewness	1.949	2.061	1.771	1.906
Kurtosis	9.763	10.270	8.259	9.124
ρ_1	0.982	0.984	0.987	0.982
ADF in Level	-4.684***	-4.464***	-4.334***	-4.325***

Summary Statistics of VIX Futures (26/03/2004 until 17/02/2012 daily)

	VIX Futures Settlement Price M1	VIX Futures Settlement Price M2	VIX Futures Settlement Price M3
Mean	21.608	22.344	22.750
Standard Deviation	9.895	8.831	8.068
Skewness	1.696	1.326	1.124
Kurtosis	6.366	4.996	4.371
ρ_1	0.990	0.992	0.995
ADF in Level	-2.696*	-2.282	-2.054
ADF in First Difference	-8.341***	-9.198***	-9.272***

Summary Statistics of VSTOXX (04/01/1999 until 24/02/2012 daily)

	VSTOXX
Mean	26.388
Standard Deviation	8.249
Skewness	1.380
Kurtosis	5.401
ρ_1	0.984
ADF in Level	-3.940***

Summary Statistics of VSTOXX Futures (30/04/2009 until 09/02/2012 daily)

	VSTOXX Futures Close Price M1	VSTOXX Futures Close Price M2	VSTOXX Futures Close Price M3
Mean	24.478	24.071	22.429
Standard Deviation	11.014	11.764	13.136
Skewness	-0.894	-1.100	-0.895
Kurtosis	3.814	3.305	2.315
ρ_1	0.859	0.786	0.782
ADF in Level	-2.900**	-2.988**	-2.479
ADF in First Difference	-8.830***	-13.053***	-9.402***

*Notes: The optimum number of lags used in the ADF test equation is based on AIC. *, **, and *** denote significance at the 10%, 5% and 1% level respectively. ρ_1 is first order autocorrelation that is derived using the Correlogram.*

Appendix B Scatterplots of returns for equity and volatility indexes

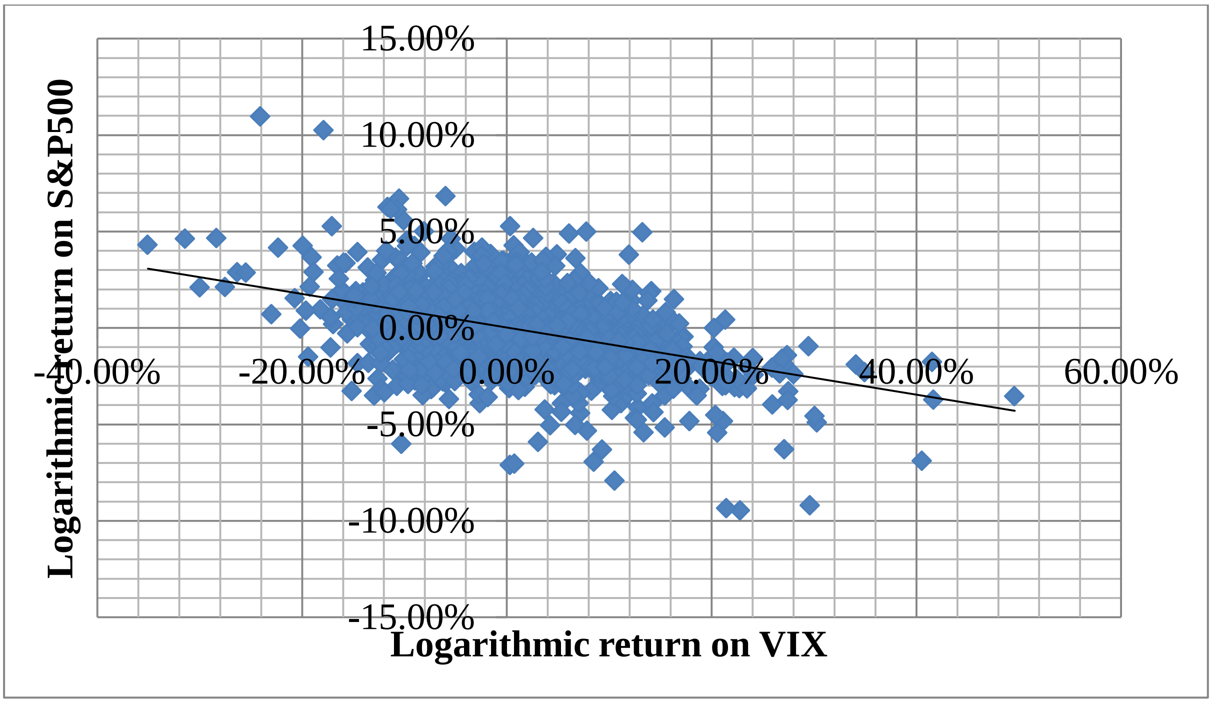

Figure A1. Scatter plot of pairs of logarithmic returns for VIX and S&P500 between 02-01-1990 and 01-03-2012.

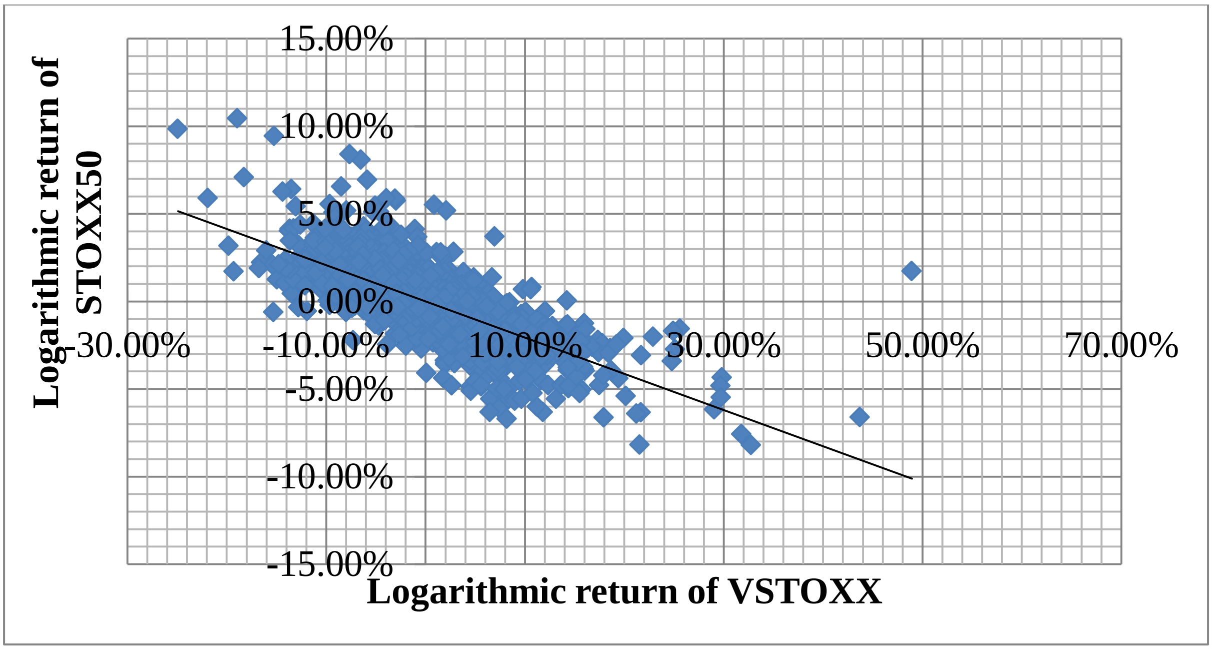

Figure A2. Scatter plot of pairs of logarithmic returns for VSTOXX and EURO STOXX 50 between 04-01-1999 and 24-02-2012.

Appendix C Portfolio Diversification with Volatility Futures – supplementary results

C.1 VIX

Summary stats - log returns, daily rebalancing, non-zero RF rate, but no collateralization of the futures						
All sample (2004- 2012)						
	S&P 500	S&P 500 2.5% VIX futures	S&P 500 10% VIX futures	S&p 500 Bonds	S&P 500 Bonds VIX Futures 2.5%	S&P 500 Bonds VIX Futures 10%
Mean return	2.63%	2.60%	2.51%	3.65%	3.59%	**3.42%**
Volatility	22.27%	20.47%	16.03%	13.02%	11.51%	**8.96%**
Min	-9.47%	-8.81%	-7.14%	-5.72%	-5.15%	-3.84%
Max	10.96%	10.40%	8.73%	6.57%	6.13%	4.78%
Annual Sharpe ratio	5.90%	6.27%	7.41%	17.86%	19.71%	**23.40%**
VaR 1%(historical)	4.59%	4.25%	3.19%	2.56%	2.22%	**1.57%**
VaR 5% (historical)	2.16%	1.99%	1.39%	1.23%	1.05%	**0.68%**
subperiod 1: 2004 - 2007						
Mean return	7.57%	7.47%	7.16%	6.16%	6.10%	5.89%
Volatility	12.10%	10.86%	8.86%	7.23%	6.23%	**6.34%**
Min	-3.53%	-2.80%	-1.98%	-1.93%	-1.44%	-1.89%
Max	2.88%	2.54%	3.21%	1.82%	1.37%	3.37%
Annual Sharpe ratio	4.21%	4.78%	6.23%	25.26%	28.99%	**27.40%**
VaR 1%(historical)	2.35%	2.12%	1.41%	1.22%	1.01%	**0.78%**
VaR 5% (historical)	1.27%	1.12%	0.81%	0.76%	0.65%	**0.45%**
subperiod 2: 2008-2012						
Mean return	-1.85%	-1.82%	-1.72%	1.36%	1.31%	1.16%

Volatility	28.53%	26.33%	20.48%	16.62%	14.76%	**10.81%**
Min	-9.47%	-8.81%	-7.14%	-5.72%	-5.15%	-3.84%
Max	10.96%	10.40%	8.73%	6.57%	6.13%	4.78%
Annual Sharpe ratio	-7.49%	-8.00%	-9.80%	6.40%	6.91%	**8.11%**
VaR 1%(historical)	5.38%	4.95%	3.90%	3.11%	2.77%	**1.84%**
VaR 5% (historical)	2.92%	2.70%	1.90%	1.62%	1.37%	**0.94%**
short crisis subperiod: 2008						
Mean return	-50.80%	-47.89%	-39.14%	-28.30%	-25.94%	**-18.89%**
Volatility	41.41%	38.70%	31.11%	24.36%	22.10%	**16.40%**
Min	-9.47%	-8.81%	-7.14%	-5.72%	-5.15%	-3.84%
Max	10.96%	10.40%	8.73%	6.57%	6.13%	4.78%
Annual Sharpe ratio	-125%	-126%	-129%	-120%	-122%	**-121%**
VaR 1%(historical)	8.60%	8.05%	6.62%	5.16%	4.69%	**3.40%**
VaR 5% (historical)	4.63%	4.63%	4.63%	4.63%	4.63%	**4.63%**

Summary stats - relative returns, daily rebalancing, non-zero risk-free rate, but no collateralization of the futures

All sample (2004- 2012)	S&P 500	S&P 500 2.5% VIX futures	S&P 500 10% VIX futures	S&p 500 Bonds	S&P 500 Bonds VIX Futures 2.5%	S&P 500 Bonds VIX Futures 10%
Mean return	5.11%	5.82%	7.96%	5.16%	5.87%	**8.01%**
Volatility	22.25%	20.44%	16.06%	13.00%	11.49%	**9.10%**
Min	-9.03%	-8.37%	-6.68%	-5.46%	-4.86%	-3.71%
Max	11.58%	11.02%	9.35%	6.95%	6.51%	5.18%
Annual Sharpe ratio	17.04%	22.03%	41.33%	29.56%	39.63%	73.46%
VaR 1%(historical)	4.49%	4.08%	3.06%	2.50%	2.14%	**1.43%**

VaR 5% (historical)	2.14%	1.96%	1.35%	1.22%	1.03%	**0.66%**
subperiod 1: 2004 - 2007						
Mean return	8.30%	8.82%	10.36%	6.62%	7.18%	**8.85%**
Volatility	12.09%	10.86%	9.03%	7.23%	6.23%	6.63%
Min	-3.47%	-2.65%	-1.90%	-1.89%	-1.40%	-1.59%
Max	2.92%	2.59%	3.95%	1.84%	1.41%	4.11%
Annual Sharpe ratio	24.97%	34.79%	66.93%	46.58%	65.98%	**95.94%**
VaR 1%(historical)	2.33%	2.09%	1.37%	1.20%	0.99%	**0.76%**
VaR 5% (historical)	1.27%	1.11%	0.78%	0.76%	0.65%	**0.49%**
subperiod 2: 2008-2012						
Mean return	2.21%	3.10%	5.77%	3.84%	4.69%	**7.24%**
Volatility	28.50%	26.28%	20.46%	16.60%	14.73%	**10.88%**
Min	-9.03%	-8.37%	-6.68%	-5.46%	-4.86%	-3.71%
Max	11.58%	11.02%	9.35%	6.95%	6.51%	5.18%
Annual Sharpe ratio	6.74%	10.70%	26.81%	21.37%	29.86%	**63.88%**
VaR 1%(historical)	5.24%	4.80%	3.67%	3.03%	2.68%	**1.74%**
VaR 5% (historical)	2.88%	2.62%	1.85%	1.60%	1.35%	**0.90%**
short crisis subperiod: 2008						
Mean return	-42.23%	-38.41%	-26.98%	-23.08%	-19.75%	**-9.75%**
Volatility	41.37%	38.65%	31.06%	24.33%	22.06%	**16.38%**
Min	-9.03%	-8.37%	-6.68%	-5.46%	-4.86%	-3.71%
Max	11.58%	11.02%	9.35%	6.95%	6.51%	5.18%
Annual Sharpe ratio	-104%	-101%	-90%	-98%	-94%	**-65%**
VaR 1%(historical)	8.24%	7.67%	6.23%	4.94%	4.46%	**3.16%**
VaR 5% (historical)	4.52%	4.52%	4.52%	4.52%	4.52%	**4.52%**

2008: S&P and VIX futures, daily rebalancing, portfolio weights based on diagonal-VECH multivariate GARCH model

short crisis subperiod: 2008	
Mean return	8.99%
Volatility	69.02%
Min	-13.52%
Max	15.67%
Annual Sharpe ratio	11.64%
VaR 1%(historical)	11.75%
VaR 5% (historical)	8.30%

C.2 VSTOXX

Summary stats - log returns, daily rebalancing, zero RF						
All sample (2009- 2012)						
	EURO STOXX 50	97.5% EURO STOXX 50 2.5% VSTOXX futures	90% EURO STOXX 50 10% VSTOXX futures	EURO STOXX Bonds	EURO STOXX Bonds VIX Futures 2.5%	EURO STOXX Bonds VIX Futures 10%
Mean return	2.17%	1.80%	0.67%	3.17%	2.77%	1.56%
Volatility	25.49%	23.47%	18.09%	15.08%	13.36%	**9.57%**
Min	-6.54%	-6.01%	-5.35%	-3.91%	-3.41%	-3.19%
Max	9.85%	9.19%	7.24%	6.34%	5.77%	4.08%
Annualized Sharpe ratio	8.53%	7.66%	3.71%	20.99%	20.69%	**16.34%**
VaR 1%(historical)	4.37%	3.96%	2.77%	2.73%	2.14%	**1.40%**
VaR 5% (historical)	2.59%	2.35%	1.83%	1.59%	1.36%	**0.91%**

Summary stats -log returns, daily rebalancing, with non-zero risk-free rate but no collateralization of the futures						
All sample (2009- 2012)						
	EURO STOXX 50	97.5% EURO STOXX 50 2.5% VSTOXX futures	90% EURO STOXX 50 10% VSTOXX futures	EURO STOXX Bonds	EURO STOXX Bonds VIX Futures 2.5%	EURO STOXX Bonds VIX Futures 10%
Mean return	2.17%	1.80%	0.67%	3.17%	2.77%	1.56%
Volatility	25.49%	23.47%	18.09%	15.08%	13.36%	9.57%
Min	-6.54%	-6.01%	-5.35%	-3.91%	-3.41%	-3.19%
Max	9.85%	9.19%	7.24%	6.34%	5.77%	4.08%
Annualized Sharpe ratio	-6.80%	-8.98%	-17.89%	-4.92%	-8.55%	-24.48%
VaR 1%(historical)	4.37%	3.96%	2.77%	2.73%	2.14%	1.40%
VaR 5% (historical)	2.59%	2.35%	1.83%	1.59%	1.36%	0.91%

Summary stats -relative returns, daily rebalancing, with non-zero risk-free rate but no collateralization of the futures						
All sample (2009- 2012)						
	EURO STOXX 50	97.5% EURO STOXX 50 2.5% VSTOXX futures	90% EURO STOXX 50 10% VSTOXX futures	EURO STOXX Bonds	EURO STOXX Bonds VIX Futures 2.5%	EURO STOXX Bonds VIX Futures 10%
Annualized Mean return	5.42%	5.73%	6.65%	5.13%	5.45%	**6.39%**
Volatility (annualized) St dev	25.51%	23.49%	18.12%	15.10%	13.37%	**9.66%**
Min	-6.33%	-5.81%	-5.21%	-3.79%	-3.25%	-3.10%

Max	10.35%	9.71%	7.81%	6.64%	6.10%	4.48%
Annualized Sharpe ratio	5.92%	7.74%	15.12%	8.11%	11.51%	**25.71%**
VaR 1%(historical)	4.28%	3.86%	2.69%	2.67%	2.07%	**1.38%**
VaR 5% (historical)	2.55%	2.32%	1.78%	1.57%	1.34%	**0.88%**

Summary stats -relative returns, daily rebalancing, with non-zero risk-free rate, with collateralization of the futures

All sample (2009- 2012)	EURO STOXX 50	97.5% EURO STOXX 50 2.5% VSTOXX futures	90% EURO STOXX 50 10% VSTOXX futures	EURO STOXX Bonds	EURO STOXX Bonds VIX Futures 2.5%	EURO STOXX Bonds VIX Futures 10%
Annualized Mean return	5.42%	5.83%	7.04%	5.13%	5.55%	**6.79%**
Volatility (annualized) St dev	25.51%	23.49%	18.12%	15.10%	13.37%	**9.66%**
Min	-6.33%	-5.81%	-5.21%	-3.79%	-3.25%	-3.10%
Max	10.35%	9.71%	7.82%	6.64%	6.10%	4.48%
Annualized Sharpe ratio	5.92%	8.16%	17.30%	8.11%	12.25%	**29.79%**
VaR 1%(historical)	4.28%	3.86%	2.69%	2.67%	2.07%	**1.38%**
VaR 5% (historical)	2.55%	2.32%	1.78%	1.57%	1.34%	**0.88%**

Appendix D: GARCH Models

Model Name	Variance Model Specification	Condition for finite unconditional variance
AR(4)-N-GARCH(1,1)	$y_t = c_0 + c_1 y_{t-1} + c_2 y_{t-2} + c_4 y_{t-4} + \varepsilon_t$ $\varepsilon_t = z_t \sigma_t$ $z_t \sim N(0,1)$ $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$	$\alpha + \beta < 1$
AR(4)-T-GARCH(1,1)	$y_t = c_0 + c_1 y_{t-1} + c_2 y_{t-2} + c_4 y_{t-4} + \varepsilon_t$ $\varepsilon_t = z_t \sigma_t$ $z_t \sim Student - t(0,1)$ $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$	$\alpha + \beta < 1$
AR(4)-N-GJR	$y_t = c_0 + c_1 y_{t-1} + c_2 y_{t-2} + c_4 y_{t-4} + \varepsilon_t$ $\varepsilon_t = z_t \sigma_t$ $z_t \sim N(0,1)$ $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \lambda\varepsilon_{t-1}^2 1(\varepsilon_{t-1} < 0)$ $1(\varepsilon_t < 0) = \begin{cases} 1, \text{if } \varepsilon_t < 0 \\ 0, \text{otherwise} \end{cases}.$	$\alpha + \beta + \frac{\lambda}{2} < 1$

AR(4)-T-GJR	$y_t = c_0 + c_1 y_{t-1} + c_2 y_{t-2} + c_4 y_{t-4} + \varepsilon_t$ $\varepsilon_t = z_t \sigma_t$ $z_t \sim Student - t(0,1)$ $\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \lambda \varepsilon_{t-1}^2 1(\varepsilon_{t-1} < 0)$ $1(\varepsilon_t < 0) = \begin{cases} 1, \text{ if } \varepsilon_t < 0 \\ 0, \text{ otherwise} \end{cases}.$	$\alpha + \beta + \frac{\lambda}{2} < 1$
AR(4)-N-EGARCH	$y_t = c_0 + c_1 y_{t-1} + c_2 y_{t-2} + c_4 y_{t-4} + \varepsilon_t$ $\varepsilon_t = z_t \sigma_t$ $z_t \sim N(0,1)$ $\ln(\sigma_t^2) = \omega + \alpha \left[\frac{\lvert \varepsilon_{t-1} \rvert}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] + \beta \ln(\sigma_{t-1}^2) + \lambda \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}$	Not applicable (variance always converges to a finite long term mean)
AR(4)-T-EGARCH	$y_t = c_0 + c_1 y_{t-1} + c_2 y_{t-2} + c_4 y_{t-4} + \varepsilon_t$ $\varepsilon_t = z_t \sigma_t$ $z_t \sim Student - t(0,1)$ $\ln(\sigma_t^2) = \omega + \alpha \left[\frac{\lvert \varepsilon_{t-1} \rvert}{\sqrt{\sigma_{t-1}^2}} - E \left[\frac{\lvert \varepsilon_{t-1} \rvert}{\sqrt{\sigma_{t-1}^2}} \right] \right] + \beta \ln(\sigma_{t-1}^2) + \lambda \frac{\varepsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}}$	$\alpha + \lvert \gamma \rvert \geq 0$